Magnetizing Current And Transformer Design Optimization

by Dennis Feucht, Innovatia Laboratories, Cayo, Belize

Transformer design is often optimized by filling the winding window with windings to minimize winding loss. Magnetizing current is negligible with maximum winding inductance from maximum turns, resulting in negligible core loss from magnetizing-current ripple. These conditions usually prevail for transformer applications having high primary voltage and low current, which is high input-resistance R_g design to which textbook transformer models largely apply.

However, there is a trend toward low-converter-port resistance R_g in digital logic "point of load" (POL) and battery-sourced converters. This article is about a different category of low-R_g power-transfer circuits, those with transformers, and addresses transformer design optimization for such circuits.

We begin with a comparison of inductor and transformer design for power conversion, analyzing how ripple and therefore magnetizing current is a factor in low-R_g transformers. Since it cannot be eliminated, this creates an opportunity to use magnetizing current to optimize transformer designs through its impact on three design criteria—core utilization, power transfer, and operating frequency. The principles for optimizing these design criteria are explained with further details of implementation provided in the references.

Considering Ripple In Low-R_g Transformer Designs

In inductor design, a major goal is to maximize magnetic energy storage in the core so that it is fully utilized. This occurs when the circuit drives the core to its full power-loss and saturation values.[1] However, the function of a transformer is not to *store* but to *transfer* energy from primary to secondary winding(s). Ideally, no storage occurs in a transformer, while in an inductor, power transfer through intermediate storage is its purpose.

Converter inductor current waveforms usually differ greatly from transformer waveforms in that inductor current has *ripple factor*

$$\gamma = \frac{\Delta i / 2}{\bar{i}} = \frac{\hat{i}_\sim}{I} << 1$$

That is, ripple (~) amplitude (^) $\hat{i}_\sim$ of the current waveform is negligible relative to average current $\bar{i}$, the small-ripple assumption, $\hat{i}_\sim << \bar{i}$ or $\gamma << 1$ applies, and the current can be approximated as constant.

Coupled-inductor winding fields aid, the field flux of the windings do not cancel, and saturation and core loss driven by magnetic flux are major design considerations. In contrast, transformer winding current waveforms are bipolar, and for each polarity or half-cycle, $\gamma = 1$. (Bipolar waveform $\gamma \to \infty$ is meaningless.) Contrasted with typical inductor use, transformer winding currents are bipolar and symmetric; they are all ripple. Each half-cycle, $\gamma = ½/D$, where D = duty-ratio. Square-waves with $D = ½$ have $\gamma = 1$.

During each half-cycle of a typical switching power-transfer circuit, a voltage square-wave is applied to the primary winding. The magnetizing current in the finite transformer winding inductance L_p adds an upward or downward sloping ramp to the square-wave current waveform levels. Because primary and secondary winding fields oppose and cancel, the net ripple flux in the core is the magnetizing flux. It is usually small and is ignored. However, for low-R_g transformer designs, the magnetizing current can be significant, and the

$$\text{small-ripple approximation: } \hat{i}_\sim << \bar{i} \Rightarrow \gamma << 1$$

cannot be assumed. Following the engineering adage that "If you can't fix it, feature it" instead of ignoring the magnetizing current, we will optimize its involvement in transformer power transfer.

With the low port resistance of computer boards or battery converters, the textbook view of transformer design is inadequate because magnetizing current is significant. A winding with perhaps only 2 to 5 turns trades off (in the fixed winding window) reduced turns for larger wire size (conductor area) to conduct high currents. This results in low inductance and high current ripple. The design goal of *maximum power transfer* from input to output port through the transformer is then achieved by including three design criteria in the transformer's design optimization.

First Criterion: Full Core Utilization

The first optimizing criterion is the same as for inductors, and it applies to the magnetizing component of winding current. With few turns N, wire size and current are large, and the core is operated at its magnetic limits.

As shown in reference [2], the minimum turns N_λ allowed by core loss and N_i allowed by saturation together determine the minimum N for transformers. Whichever is greater sets the minimum N, and N is minimized when they are equal: $N = N_\lambda = N_i$. This is the same condition that maximizes inductor power density and allows both field flux ϕ and field current Ni to be driven simultaneously to their maximum values, minimizing core size.

For transformer design, the bounds on N are consequently

$$\max\{N_\lambda, N_i\} \le N \le N_w$$

where N_w is the maximum turns that fit the window winding area allotted for the given winding.

In contrast, for inductors, the design range for turns is

$$N_\lambda \le N \le \min\{N_i, N_w\}$$

and the lesser of saturation N_i or maximum window turns N_w sets the maximum N.

So for inductors, the optimal N is $N_{opt} = N_\lambda = N_i$; the allowable range collapses to a single optimal value and both core loss (from N_λ) and saturation (from N_i) limits are at their optimal ratio. Then when either is driven to its maximum at full-scale power, so is the other.

When the small-ripple approximation does not apply, there is no longer separation of variables between core power loss from magnetic field ripple ΔB and saturation from average field intensity $\overline{H}$. Whenever $\gamma << 1$, ripple is negligible and average and peak current, and hence field intensity H, are about equal; $\hat{H} \approx \overline{H}$, and the $\overline{H}$ static magnetic operating-point of the core can be related to fractional saturation k_{sat} as plotted on core catalog saturation graphs.

However, for $\gamma \to 1$, the peak value of H or B must also be considered because it significantly exceeds the average. For triangle-wave magnetizing currents, the peak values are twice the average values in each half-cycle, half-cycle $\gamma = 1$, and the operating-point is set at half the peak value, or $\overline{B} = \hat{B}/2$, and $\Delta B = \hat{B}$.

The design goal is to maximize *transfer power* stored and released by the core from magnetizing current for a total current waveform having $\gamma = 1$ and limited by core saturation to a peak current $\hat{i}$. The current waveform cannot exceed this peak value while maximizing linear core energy density

$$\Delta w_L = \Delta\phi \cdot N\bar{i} = (\Delta B \cdot A) \cdot (N \cdot \bar{i})$$

where A = magnetic path area, $\Delta\phi$ = the change in field flux, and $N\bar{i}$ = the average field current.

To maximize Δw_L by adjusting both ripple and average current, constrained by peak value $\hat{i}$, at what ratio of ripple to average is the stored energy maximized? It is at $\gamma = 1$, or when $\hat{i} = 2 \cdot \bar{i}$.[3] Therefore, core materials with a high optimal γ ($\gamma_{opt} \to 1$) are more fully utilized in transformers than in inductors. That is why ferrites and not powdered-iron cores are chosen for transformers.

Second Criterion: Maximum Power Transfer Across Windings

The second criterion is *power transfer*, maximizing the fraction of primary winding power transferred to the secondary winding(s)—the same as transformer efficiency:

$$\eta = \frac{\overline{P}_s}{\overline{P}_p}$$

The usual assumption is that the *maximum power transfer theorem* from circuit theory applies (which might also be called the "maximum output power theorem"), but it is only an approximation for a transformer model because of the core loss represented by shunt resistance R_c between the primary and secondary winding resistances, as shown in Fig. 1.[4] R_c is not included in the single-loop circuit of the maximum power-transfer theorem.

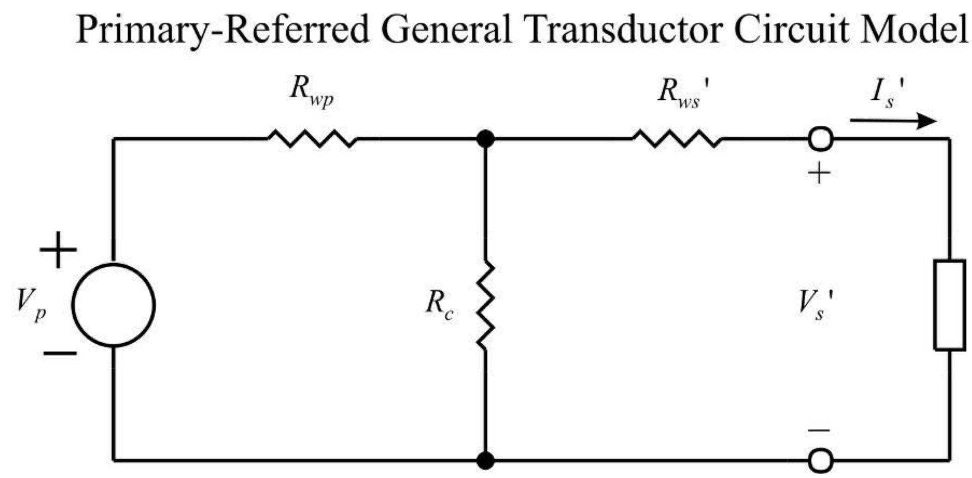

Fig. 1. Transformer equivalent circuit model.

The winding resistances are primary resistance R_{wp} and secondary winding resistance referred to the primary circuit R_{ws}'. R_{wp} and R_{ws}' cannot be combined without invalidating the model. Core loss is in the core equivalent resistance R_c which, if low enough in value, causes max η to be at other than where winding and core loss are equal.

For an ideal transformer with no core loss, $R_c \to \infty$, the winding resistances can be combined into a single R_w, and the textbook model becomes valid. But for significant magnetizing current and relatively low R_c, the transfer efficiency curves appear as in Fig. 2[4] for the simplified (one-sided) secondary-referred transformer circuit model in Fig. 3.

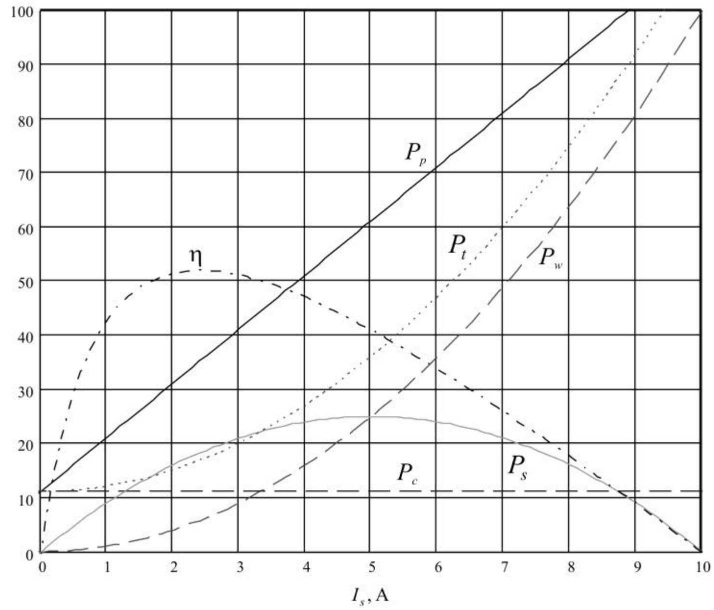

Fig. 2. Plots of average primary winding power P_p, secondary power P_s, winding and core power losses P_w and P_c, total loss P_t, and transfer efficiency $\eta = P_s / P_p$, where $P_p = P_s + P_t$ for secondary-referred model shown in Fig. 3.

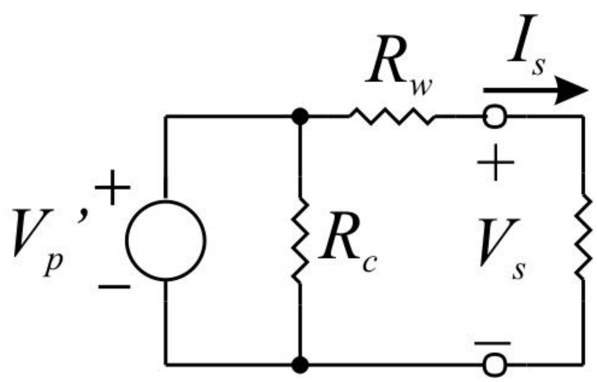

Fig. 3. Secondary-referred single-sided model with lumped R_w. $V_p' = V_p / n$ - V_p referred to the secondary winding.

Even in this simplified case having a single, lumped winding resistance R_w, maximum η does not occur for equal load resistance and R_w. The circuit is still not the same as that of the textbook theorem because of R_c. The design goal of maximizing output power requires a different and more complicated solution.[5] Core loss with R_c affects power transfer optimization, and R_c becomes part of the design procedure.

Simply put, given that magnetizing current and power are unavoidably significant, they are included in power transfer by maximizing the amount transferred by them. The optimization criteria are similar to those of inductors because both store energy in the core.

Third Criterion: Frequency

Inductors have an optimal, maximized frequency that maximizes core transfer power by transferring more quickly the per-cycle energy the core stores. The limit is where the increase in core power loss exceeds power transferred, at f_{MAX}.[6] This does not ordinarily apply to transformers because power is transferred directly, from winding to winding, at whatever frequency within some broad range. Transformers are limited by frequency in that the magnetizing current, i_m can cause excessive core loss, though the higher the frequency, the less is the ripple magnitude, $\hat{i}_{m\sim} = \Delta i_m / 2$.

At some frequency, where ripple and frequency combine as large enough to overheat the core, then frequency is a design factor. While ripple amplitude is reduced with increasing frequency, core loss is not linearly related to frequency; in the generalized Steinmetz model of core power loss, it varies by a frequency exponent greater than one, and core loss increases at a higher rate with frequency than ripple decreases. Large magnetizing-current ripple Δi_m can cause core power-loss or saturation limits to be exceeded.

Also, at the low end of the frequency range, as magnetizing-current ripple amplitude $\hat{i}_{m\sim}$ increases with decreasing frequency, it becomes too large and either overheats the core or saturates it excessively on peaks. Consequently, there is an acceptable frequency range for transformer design to avoid exceeding core limits.

Summary

For low-R_g transformer applications, magnetizing current cannot usually be ignored. The optimization criteria for magnetizing current are related to inductor optimization because magnetizing current stores energy in the transformer core. If it cannot be made negligible, then conditions in the magnetic design—that is, the choice of core geometry and material, and primary turns—can maximize its use instead.

Furthermore, as core loss is higher because of large magnetizing current, this causes core resistance R_c in the transformer circuit model to be lower in value and increases the importance of criteria that maximize winding-to-winding power transfer. And thirdly, there is an optimal range of frequency for driving the transformer that trades off maximizing magnetizing energy transfer with minimizing core loss. This is an optimization topic in itself.

The references contain the fundaments with more details for carrying out these optimizations in a design, and if you are intending to use this overview as a design strategy, I recommend that you print out and read the references. Together, along with this article, they comprise a viewpoint on transformer magnetics optimization that includes aspects of design that might otherwise be neglected.

References

1.“Utilizing Full Saturation and Power Loss To Maximize Transfer Power In Magnetic Components” by Dennis Feucht, How2Power Today, February 2011 and in *Power Magnetics Design Optimization (PMDO)* by Dennis Feucht, “Optimal Turns”, pages 221 - 224.

2. “Transformer Design (Part 1): Maximizing Core Utilization” by Dennis Feucht, How2Power Today, January 2018; from *PMDO*, “Transformer Optimization”, pages 251 - 255, innovatia.com.

3. “Maximum Power with Large Ripple,” *PMDO*, pages 247 - 251, innovatia.com.

4. “Magnetics Optimization (Part 1): Equal Core And Winding Losses Do Not Maximize Power Transfer” by Dennis Feucht, How2Power Today, September 2015; in *PMDO*, “General Power-Transfer Circuit Model”, “Core and Winding Power Loss”, “Efficiency”, “Power-Loss ratio”, and “Optimal Voltage and Power-Loss Ratios”, pages 289 - 299.

5. “Magnetics Optimization (Part 3): Maximum Power Transfer Of Magnetics Circuit Model” by Dennis Feucht, How2Power Today, April 2017. Also see, *PMDO* in [4].

6. “Determining Maximum Usable Switching Frequency For Magnetics In CCM-Operated Converters” by Dennis Feucht, How2Power Today, April 2015 and in *PMDO*, “Frequency Optimization,” pages 255 - 260.

About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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