Inductor Turns For Maximum Energy Transfer With Core Saturation

by Dennis Feucht, Innovatia Laboratories, Cayo, Belize

In a previous article,[1] an asymptotic semi-log model of core saturation was presented. The saturation model is shown in Fig. 1. It has three regions of core operation: unsaturated ($H < H_0$), saturated ($H_0 \le H < H_T$), and fully-saturated ($H \ge H_T$). Power inductors are operated in the saturated region because it is in this region that maximum energy or flux can be stored.

This article applies the asymptotic core saturation model to the main magnetics design goal of maximizing energy density in the core for power transfer. This basic performance goal optimizes inductor turns. To do this, we must determine the value of k_{sat} and H (representing average current) in the core saturation model where core transfer-energy density is maximized to achieve maximum power transfer in a power converter for a given core size. Since circuit design parameters determine average current and can affect the minimum allowable k_{sat}, the finding of the optimum k_{sat} may lead the designer to select a larger or smaller core.

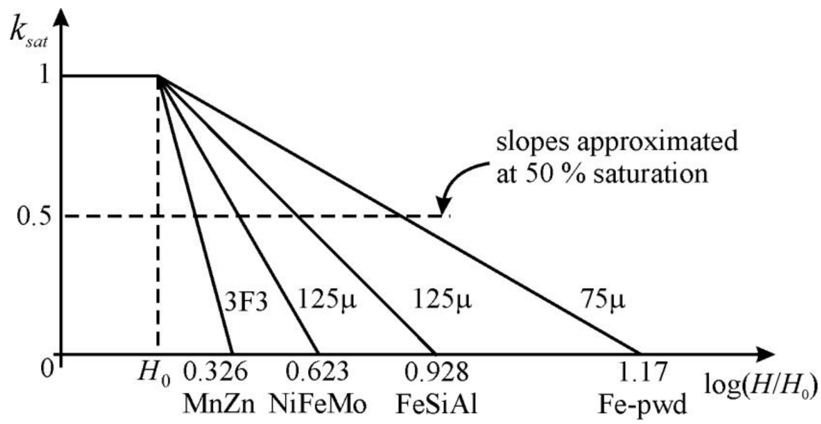

Fig. 1. Semi-log asymptotic approximation showing the saturation region of $k_{sat}(H)$, normalized to H_0 for various materials. The "75µ" is Micrometals 26 material. The values of H_T for each material are shown on the x axis.

The Core Saturation Model

The asymptotic plots are based on

$$k_{sat} = 1 - \frac{\log H - \log H_0}{\log H_T - \log H_0} = 1 - \frac{\log(H/H_0)}{\log(H_T/H_0)} = \frac{\log(H_T/H)}{\log(H_T/H_0)} \quad , H \ge H_0$$

where H_T is the value of H at which the line segment intersects zero ($k_{sat} = 0$), and H_0 is the asymptotic breakpoint at the onset of saturation. H_0 can be found on the graph of vendor-supplied curves for k_{sat} by linearly extrapolating a line tangent to the plot at $k_{sat} = 0.5$ upward to where it intersects $k_{sat} = 1$, at H_0, as shown in Fig. 2 on the catalog curve for Micrometals 26 material. For $H < H_0$, $k_{sat} = 1$.

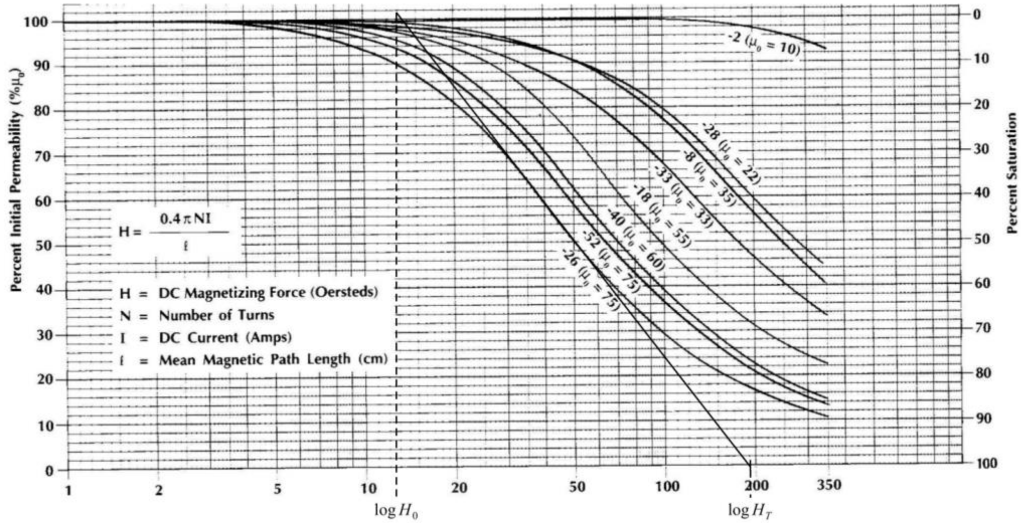

Fig. 2. Linearized approximation at k_{sat} = 0.5 of saturation region, drawn on catalog curve for Micrometals 26 material.

What Is The Optimal Core Saturation?

In previous articles[2,3] it was shown that the allowable turns N have a range between core power loss at minimum $N = N_\lambda$ (which corresponds to the maximum allowable core loss) and the saturation limit at maximum $N = N_i$. However, it was left to the designer to choose how far the core is allowed to be driven into saturation, as quantified in reference [2] by the *saturation factor*

$$k_{sat} = \frac{\mu}{\mu_i} = \frac{\mathcal{L}}{\mathcal{L}_0} = \frac{L(i)}{L(0\,\mathrm{A})}$$

where μ = permeability, L = circuit inductance, and $\mathcal{L}$ is L referred to the magnetic field as *field inductance*, $\mathcal{L} = L/N^2$.

The saturation factor k_{sat} decreases from one at zero current. The quantities in the denominators are the zero-current (unsaturated) values. Circuit current i at the inductor terminals is referred to the field as *field current* $Ni = N \cdot i$ that increases with N, causing field intensity H to increase proportionally. Circuit inductance is related to k_{sat} by

$$L = k_{sat} \cdot L(0\,\mathrm{A}) = k_{sat} \cdot L_0 = N^2 \cdot (k_{sat} \cdot \mathcal{L}_0)$$

As turns decrease, the field ripple amplitude $\hat{B}_{\sim}$, found on the horizontal axis of core catalog power-loss graphs in the materials section, increases until the power-loss limit is reached at

$$N_\lambda = \frac{\Delta\lambda}{\Delta\phi_\lambda} = \frac{\Delta\lambda}{\Delta\phi(\overline{p}_c)} = \frac{\Delta\lambda}{\Delta B \cdot A} = \frac{\Delta\lambda}{(2 \cdot \hat{B}_{\sim}) \cdot A}$$

where A is the core magnetic cross-sectional area, $\overline{p}_c$ is the maximum core power-loss density, and $\Delta\lambda = V_L \cdot \Delta t$ is the circuit flux (across inductor terminals) of duration Δt. The field flux ripple $\Delta\phi_\lambda$ is the $\Delta\phi$ corresponding to maximum allowable power loss in the core which depends on core size and thermal characteristics.

The upper limit on N is either saturation or winding window area. For coupled inductors operating with ripple current much less than average current, the *average field current* $N\overline{i} = N \cdot \overline{i} = NI$ is the magnetic operating-point NI. At the maximum allowable saturation, or minimum k_{sat},

$$N_i = \frac{N\bar{i}(k_{sat})}{\bar{i}} = \frac{NI(k_{sat})}{I}$$

where $\bar{i} = I$ is the average winding current corresponding to k_{sat} and (by Ampere's Law), $N\bar{i}(k_{sat}) = \overline{H}(k_{sat}) \cdot l$, where l is the core magnetic path length and $\overline{H}(k_{sat})$ is the average field intensity as plotted on saturation graphs in core catalogs. (The plots are actually given by core manufacturers, as in Fig. 2, as $k_{sat}(\overline{H})$, with k_{sat} on the vertical axis of the graph.)

Thus, the design range of N is between these limits;

$$N_\lambda \le N \le \min\{N_i, N_w\}$$

where N_w is the number of turns that fill the winding window.

For maximum power transfer through the core—that is, for full core *utilization*—power-loss and saturation are increased to the maximum allowable values by increasing to full-scale $\Delta\lambda$ which increases N_λ; and average current is increased to full-scale which decreases N_i until optimum turns $N_{opt} = N_\lambda = N_i$ at full core utilization. Then maximum power is transferred by the core as

$$\overline{P}_L = \Delta\phi_\lambda \cdot N\bar{i}(k_{sat}) \cdot f$$

where f is the frequency of the waveform driving the core. This power equation is derived by setting equal the expressions $N_\lambda = N_i$ and solving for power.

This optimization of turns does not result in a deterministic procedure because an optimal choice for k_{sat} is lacking. What is its optimal value?

Our goal therefore is to complete the procedure by finding the value of k_{sat} where core transfer-energy density is maximized so that for a given core size, maximum power can be transferred between power ports of the converter.

Note that we are also concerned with average current, which corresponds to H on the horizontal axis of the saturation model—both k_{sat} and average current are equally important. However, the latter will be determined by circuit design and the resulting k_{sat} is checked in circuit design so that it not causing current waveforms to be excessively superlinear. Thus, the determination of optimum k_{sat} in an application may lead the designer to pick a larger or smaller core.

The typical inductor operating range of k_{sat} in power applications is 0.8 to 0.5. As turns are increased, inductance increases by N^2, but saturation also increases and k_{sat} decreases.

It was shown in reference [1] that N maximizes L at an optimal value of $N_{\max}$. Increasing saturation progressively decreases L with increasing turns (as Ni increases) while the turns themselves increase L. If the saturation curve decreases faster than N^2, then adding more turns decreases inductance. The operating-point of maximum L has

$$k_{sat} = \frac{\log(Ni_T / Ni)}{\log(H_T / H_0)}$$

By Ampere's Law, $H \cdot l = Ni = N \cdot i$, and $Ni_T = H_T \cdot l$. For a given core, l is constant and Ni is proportional to H. Ratios of Ni are equal to ratios of corresponding H. Thus we can express $k_{sat}(H)$ as $k_{sat}(Ni)$ and relate saturation to field current. This is more useful for optimization because it includes both design parameters N and circuit current i.

For a given $k_{sat}(H)$, H_T and H_0 are parameters of k_{sat}, and $\log(H_T/H_0)$ is constant for a given curve. Corresponding current parameters to those of H are

$$I_T = \frac{H_T \cdot l}{N}, \quad I_0 = \frac{H_0 \cdot l}{N}$$

and depend on core size for l.

$N_{\max}$ maximizes L at current operating-point I where

$$L = N^2 \cdot (k_{sat} \cdot \mathcal{L}_0) = N^2 \cdot \mathcal{L}_0 \cdot \frac{\log(Ni_T / N \cdot I)}{\log(H_T / H_0)}$$

Take the derivative dL/dN, set to zero, and solve for $N_{\max}$. The $N_{\max}$ equation reduces to

$$\boxed{N_{\max} = \frac{Ni_T}{I \cdot \sqrt{e}} \approx 0.6065 \cdot \frac{Ni_T}{I} = 0.6065 \cdot \frac{H_T \cdot l}{I}}$$

For a given core, $Ni_T = H_T \cdot l$ is constant, and $N_{\max}$ varies inversely with I. Maximum inductance is found by substituting $N_{\max}$ into the inductance equation and reducing;

$$\boxed{L_{\max} = \left(\frac{H_T \cdot l}{I}\right)^2 \cdot \frac{\mathcal{L}_0}{\log(H_T / H_0)} \cdot \frac{\log \sqrt{e}}{e} \approx \left(\frac{H_T \cdot l}{I}\right)^2 \cdot \frac{\mathcal{L}_0}{\log(H_T / H_0)} \cdot (0.07988)}$$

For a fixed N, $L_{\max}$ is inversely proportional to I^2. The saturation factor at $L_{\max}$ is found from

$$k_{sat} = \frac{L_{\max}}{L_0} = \frac{L_{\max}}{N_{\max}^2 \cdot \mathcal{L}_0} = \frac{N_{\max}^2 \cdot \mathcal{L}_0 \cdot \dfrac{\log \sqrt{e}}{\log(H_T / H_0)}}{N_{\max}^2 \cdot \mathcal{L}_0} \Rightarrow \boxed{k_{sat} = \frac{\log \sqrt{e}}{\log(H_T / H_0)} \approx \frac{0.217}{\log(H_T / H_0)}}$$

Thus k_{sat} at $L_{\max}$ is determined only by the properties of the core material.

As an example, a Micrometals T201-26 Fe-pwd core has l = 118 mm and $\mathcal{L}_0$ = 242 nH. At I = 30 A of circuit current,

$$N_{\max} \approx 0.6065 \cdot \frac{H_T \cdot l}{I} = (0.6065) \cdot \frac{(15305\ \text{A/m}) \cdot (0.118\ \text{m})}{30\ \text{A}} = (0.6065) \cdot \frac{1806\ \text{A}}{30\ \text{A}} = \frac{1095\ \text{A}}{30\ \text{A}} = 36.5$$

At N = 36.5, NI = (36.5)·(30 A) = 1095 A of static field current. At this rather high value, $H = NI/l$ = 9280 A/m (117 Oe), which on the catalog saturation graph of Fig. 2 corresponds to $k_{sat} \approx 0.26$, and for which

$$L = N^2 \cdot [k_{sat} \cdot \mathcal{L}_0] = (36.5)^2 \cdot [(0.26) \cdot (242\ \text{nH})] = 83.8\ \mu\text{H}$$

Using L derived from the saturation approximation,

$$L_{\max} \approx \left(\frac{H_T \cdot l}{I}\right)^2 \cdot \frac{\mathcal{L}_0}{\log(H_T / H_0)} \cdot (0.07988) = \left(\frac{1806\ \text{A}}{30\ \text{A}}\right)^2 \cdot (206.8\ \text{nH}) \cdot (0.07988) = 59.9\ \mu\text{H}$$

This lower approximated value results from linearly extending the catalog curve downward, reaching a lower value of $k_{sat} \approx 0.186$ than the actual curve value for the given $H(I)$. This value of k_{sat} corresponds to maximum energy storage in the core. What this example shows is that maximum inductance is achieved for a 30-A circuit current for Fe-pwd 26 material by driving it hard with many turns. At half the current of 15 A, N_{max} doubles to 73 turns, and

$$L_{max}(15\text{ A}) = (30\text{ A}/15\text{ A})^2 \cdot L_{max}(30\text{ A}) = 4 \cdot L_{max}(30\text{ A}) = 240\ \mu\text{H}$$

Turns For Maximum Core Transfer Power

The current and turns for maximum inductance are not in themselves our goal of maximizing core energy or transfer power. The basic equation for energy is

$$W = \lambda \cdot i \Rightarrow dW = i \cdot d\lambda = i \cdot (L(i) \cdot di) = (L_0 \cdot k_{sat}(i)) \cdot i \cdot di$$

where L_0 is the unsaturated L.

A graph of $\lambda(i)$ is nonlinear, and consequently static or operating-point $\lambda/i \neq d\lambda/di$. The expressions for W, N, and I at maximum W are found by deriving them three different ways. The simplest is first, based on the static W. At an operating-point of current I,

$$W = \lambda \cdot I = (L(I) \cdot I) \cdot I = L_0 \cdot k_{sat}(I) \cdot I^2 = (L_0 \cdot I^2) \cdot \frac{\log\left(\frac{H_T \cdot l}{N \cdot I}\right)}{\log(H_T / H_0)} = \mathcal{L} \cdot (NI)^2 \cdot \frac{\log\left(\frac{Ni_T}{N \cdot I}\right)}{\log(H_T / H_0)}$$

Then the maximum W is found by differentiating and setting $dW/d(NI) = 0$. This reduces to

$$2 \cdot \ln\left(\frac{Ni_T}{N \cdot I}\right) = 1 \Rightarrow N \cdot I = \frac{Ni_T}{\sqrt{e}} \approx \frac{Ni_T}{1.65} \Rightarrow N_{opt} = \frac{Ni_T}{I \cdot \sqrt{e}}$$

Comparing with N_{max} of L_{max}, not surprisingly $N_{opt} = N_{max}$. Thus, the optimal value for $N_i = N_{opt} = N_{max}$. While this completes the optimization procedure for maximum power density in the core, additional constraints need be considered.

At some maximum value of $N = N_w$, window area constrains N, and an acceptable L_{max} for the given core size and current is unrealizable without increasing core size. Cores are not usually operated below about $k_{sat} \approx 0.5$ because the increasingly superlinear current waveform destabilizes peak current control by overly sensitizing the time when the comparator switches. The result is PWM jitter of the duty-ratio, resulting in current noise. If the comparator is not fast enough—that is, has excessive time delay—then current overshoot of the comparison value might cause overcurrent circuit failure.

Another derivation of the N_{opt} formula integrates differential energy;

$$W = \int dW = \int L(i) \cdot i \cdot di = \int L_0 \cdot k_{sat}(i) \cdot i \cdot di = L_0 \cdot \int \frac{\log\left(\frac{NI_T}{N \cdot i}\right)}{\log(H_T / H_0)} \cdot i \cdot di = \frac{L_0}{\ln(H_T / H_0)} \cdot \int -\ln\left(\frac{N \cdot i}{NI_T}\right) \cdot i \cdot di$$

where $\log(x) = \ln(x)/\ln(10)$ and $\ln(x) = -\ln(1/x)$. In the next step, apply the integration formula

$$\int x \cdot \ln x \cdot dx = \tfrac{1}{2} \cdot x^2 \cdot (\ln x - \tfrac{1}{2})\ ,\ x = \frac{N \cdot i}{Ni_T} \Rightarrow$$

$$W = \frac{L_0}{\ln(H_T / H_0)} \cdot \frac{Ni_T}{N} \cdot \int -\ln\left(\frac{N \cdot i}{Ni_T}\right) \cdot \left(\frac{N \cdot i}{Ni_T}\right) \cdot \left[\left(\frac{Ni_T}{N}\right) \cdot d\left(\frac{N \cdot i}{Ni_T}\right)\right] \Rightarrow$$

$$W = \frac{\frac{1}{2} \cdot L_0 \cdot i^2}{\ln(H_T / H_0)} \cdot \frac{N}{Ni_T} \cdot \left(\frac{1}{2} - \ln\left(\frac{N \cdot i}{Ni_T}\right)\right) = a \cdot (\tfrac{1}{2} \cdot x^2 - x^2 \cdot \ln x)\,,\ a = \frac{\frac{1}{2} \cdot L_0}{\ln(H_T / H_0)} \cdot \frac{Ni_T}{Ni}\,,\ x = \frac{N \cdot i}{N_T}$$

To find the maximum W for both I and N, first find the value of N from $dW/dx = 0$;

$$\frac{dW}{dx} = a \cdot (x - 2 \cdot x \cdot \ln x - x^2 / x) = -2 \cdot a \cdot x \cdot \ln x = 0$$

The constant expression $2 \cdot a > 0$ and $x > 0$. Thus, the remaining factor must be zero. Solving,

$$-\ln x = \ln\left(\frac{1}{x}\right) = \ln\left(\frac{Ni_T}{N \cdot i}\right) = 0 \Rightarrow \frac{Ni_T}{N \cdot i} = 1 \Rightarrow N = \frac{Ni_T}{i}$$

To interpret this result, we have integrated $L(i) \cdot i \cdot di$ over the full saturation region (from H_0 to H_T) of i. $L(i) = d\lambda/di$ at i and $L(i) \cdot di$ is, graphically, strips of $d\lambda$ corresponding to di being summed as the integral. Consequently, as integration proceeds across the range of i, λ accumulates to a maximum value at Ni_T. This value of N maximizes flux, and it is maximum at Ni_T beyond which no further increase in flux occurs.

The value of N for maximum W is found instead from $dW/dN = 0$, but because N occurs in the math wherever Ni_T occurs as N/Ni_T, this expression is chosen as the independent variable. Thus, holding constant $i = I$ and reducing,

$$\frac{dW}{d(N / Ni_T)} = \frac{\frac{1}{2} \cdot L_0 \cdot I^2}{\ln(H_T / H_0)} \cdot \left(\frac{1}{2} - \ln\left(\frac{N \cdot I}{Ni_T}\right) - \frac{N}{Ni_T} \cdot \left(\frac{Ni_T}{N \cdot I} \cdot I\right)\right) = \frac{\frac{1}{2} \cdot L_0 \cdot I^2}{\ln(H_T / H_0)} \cdot \left(-\frac{1}{2} - \ln\left(\frac{N \cdot I}{Ni_T}\right)\right) = 0 \Rightarrow$$

$$\ln\left(\frac{N \cdot I}{Ni_T}\right) = -\frac{1}{2} \Rightarrow N \cdot I = \frac{Ni_T}{\sqrt{e}} \Rightarrow N = N_{opt} = \frac{Ni_T}{I \cdot \sqrt{e}} = N_{\max}$$

It is not surprising that maximum core energy has the same condition as maximum inductance—that $N_{opt} = N_{\max}$—and that this is within the saturation region at $H_T / \sqrt{e}$. The optimal operating-point drives cores deeper into saturation than is typical of magnetic design practice, and if the window area and control circuit allow it, can increase inductor power-transfer utilization.

Finally, as an additional exercise in magnetics math, the third derivation is like the first except with variables in the field (core) reference-frame. Saturation can be expressed as a field quantity;

$$k_{sat} = \frac{\mathcal{L}}{\mathcal{L}_0} = \frac{\phi}{\mathcal{L}_0 \cdot Ni} = \frac{B \cdot A}{\mathcal{L}_0 \cdot Ni} = \frac{B \cdot A}{\frac{\phi_0}{Ni_0} \cdot Ni} = \frac{B}{B_0} \cdot \frac{Ni_0}{Ni} \Rightarrow B = \left(\frac{B_0}{Ni_0}\right) \cdot k_{sat} \cdot Ni \;;\; H = \frac{Ni}{l}$$

Substituting B and H into core energy density,

$$w = \frac{W}{V} = B \cdot H = \left(\frac{B_0}{Ni_0}\right) \cdot k_{sat} \cdot \frac{Ni^2}{l} = \left(\frac{B_0}{Ni_0 \cdot l}\right) \cdot Ni^2 \cdot \frac{\ln(Ni_T / Ni)}{\ln(H_T / H_0)} = \left(\frac{B_0}{Ni_0 \cdot l} \cdot \frac{1}{\ln(H_T / H_0)}\right) \cdot Ni^2 \cdot (\ln Ni_T - \ln Ni)$$

The maximum energy density is at the value of $Ni = N{\cdot}I$ at which $dw/dNi = 0$;

$$\frac{dw}{dNi} = \left(\frac{B_0}{Ni_0 \cdot l} \cdot \frac{1}{\ln(H_T / H_0)} \right) \cdot \left[2 \cdot Ni \cdot (\ln Ni_T - \ln Ni) - Ni \right] = 0 \;\Rightarrow$$

$$2 \cdot Ni \cdot (\ln Ni_T - \ln Ni) - Ni = 0 \;\Rightarrow\; Ni = N \cdot I = \frac{Ni_T}{\sqrt{e}} \;\Rightarrow\; N = N_{opt} = \frac{Ni_T}{I \cdot \sqrt{e}} = N_{\max}$$

In closing, the asymptotic saturation model simplifies saturation calculations by reducing them to analytic form. We have taken advantage of this simplification by deriving with closed-form math the turns N_{opt} that maximizes core utilization, storing maximum energy in the core. Then by equating $N_\lambda = N_i = N_{opt} = N_{\max}$ the maximum operating condition is found for maximum transfer power through the core.[3]

This condition also constrains the circuit power-port voltage and current to an optimal port resistance. Whenever input voltage V_g has a fixed range, the needed additional free parameter is the core material because it sets H_T and Ni_T.

References

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About The Author

Dennis Feucht has been involved in power electronics for 40 years, designing motor-drives and power converters. He has an instrument background from Tektronix, where he designed test and measurement equipment and did research in Tek Labs. He has lately been working on projects in theoretical magnetics and power converter research.

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