

Laplace Transform Simplifies Analysis Of Realistic SMPS Waveforms

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When making calculations with the rectangular waveforms present in switched-mode power supplies (SMPSs), designers try to avoid the difficulties associated with the real shape of these pulses. They ignore the fact that these pulses are not actually *rectangular* as in the ideal case, but rather *trapezoidal*. Unfortunately, replacement of trapezoidal pulses with the ideal rectangular ones may adversely affect assessment of power loss since overlapping of turn-on and turn-off times of, for example, transistors in the same column of the bridge, is the main source of dynamic power loss.

While there are "artificial" methods of including the on and off times in the analysis, these may produce false results or just be very cumbersome. However, there is a simpler alternative method for performing analysis of realistic SMPS waveforms using the Laplace transform.

This method will be particularly useful for analysis of filters. When designing different kinds of filters, whether at the output of an SMPS or on the input power supply line, engineers want to know how the filter affects the pulses present in the circuits. Analysis of the interaction of the operating pulses with these filters can conveniently be done using the Laplace transform, which, along with MathCAD-15, is a good fit for this type of analysis. The author strongly recommends usage of MathCAD-15 or its derivatives as this is the only software that provides an easy-to-use symbolic analysis.

The formula that represents a series of trapezoidal pulses, was constructed by the author a couple of years ago and used in some previous publications.^[1,2] That formula provides the basis for the application of the Laplace transform discussed here.

But before getting started, it should be noted that while a pulse train may have *any* number of pulses, the Laplace transform may become too long and require more computing resources for the inverse Laplace transform when going back from the frequency domain to the time domain. However, this limitation of the software can be overcome by limiting the analysis to three or four pulses, which is sufficient to obtain accurate results. Consequently, the Laplace transform provides an easy-to-understand-and-use multiplication of transfer functions when defining the output function (signal) of a series (parallel) connection of a few electronic units like amplifiers and filters.

This article begins by presenting an expression for a realistic, trapezoidal-shaped pulse train in the time domain, and confirming its validity. It's then shown how the Laplace transform can be applied to obtain the response of a simple filter, which is then converted back to the time domain using the inverse Laplace transform. Two numerical examples are then presented using this method to obtain the pulse train at the output of a low-pass filter. Another example illustrates how the pulse train expression can be used to adjust the timing of gate-drive signals for a MOSFET half bridge.

Defining The Pulse Train And Applying The Laplace Transform

Let's define the pulse parameters as shown in the picture in Fig. 1.

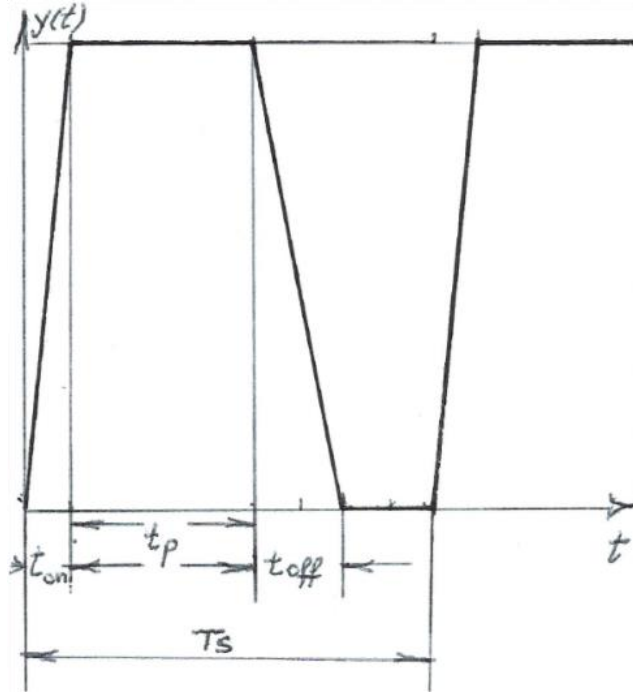


Fig. 1. We can create a pulse of an arbitrary waveform and find a waveform of the resulting signal when this pulse is pushed through a filter by applying the Laplace transform.

The pulse train in question can be described by the following expression in a symbolic form:

$$y(t) = \sum_{n=0}^{n_{\max}} \left[\Phi \left[(t - n \cdot T_s) \cdot \frac{t - n \cdot T_s}{t_{\text{on}}} - \frac{(t - t_{\text{on}} - n \cdot T_s)}{t_{\text{on}}} \cdot \Phi(t - t_{\text{on}} - n \cdot T_s) \right] \right. \\ \left. - \left(\frac{t - t_{\text{on}} - t_p - n \cdot T_s}{t_{\text{off}}} \right) \cdot \Phi(t - t_{\text{on}} - t_p - n \cdot T_s) + \frac{t - t_{\text{on}} - t_p - n \cdot T_s - t_{\text{off}}}{t_{\text{off}}} \cdot \Phi(t - t_{\text{on}} - t_p - n \cdot T_s - t_{\text{off}}) \right] \quad (1)$$

To show the expression is correct let's plot this function's graph at arbitrary values of the pulse train parameters:

$$n_{\max} = 3$$

$$T_s = 20 \mu\text{s}$$

$$t_{\text{on}} = 0.6 \mu\text{s}$$

$$t_p = 9.6 \mu\text{s}$$

$$t_{\text{off}} = 1.8 \mu\text{s}$$

$$y(t) = \sum_{n=0}^{n_{\max}} \left[\Phi \left[(t - n \cdot T_s) \cdot \frac{t - n \cdot T_s}{t_{\text{on}}} - \frac{(t - t_{\text{on}} - n \cdot T_s)}{t_{\text{on}}} \cdot \Phi(t - t_{\text{on}} - n \cdot T_s) \right] \right. \\ \left. - \left(\frac{t - t_{\text{on}} - t_p - n \cdot T_s}{t_{\text{off}}} \right) \cdot \Phi(t - t_{\text{on}} - t_p - n \cdot T_s) + \frac{t - t_{\text{on}} - t_p - n \cdot T_s - t_{\text{off}}}{t_{\text{off}}} \cdot \Phi(t - t_{\text{on}} - t_p - n \cdot T_s - t_{\text{off}}) \right]$$

The pulse train graph is then as shown in Fig. 2.

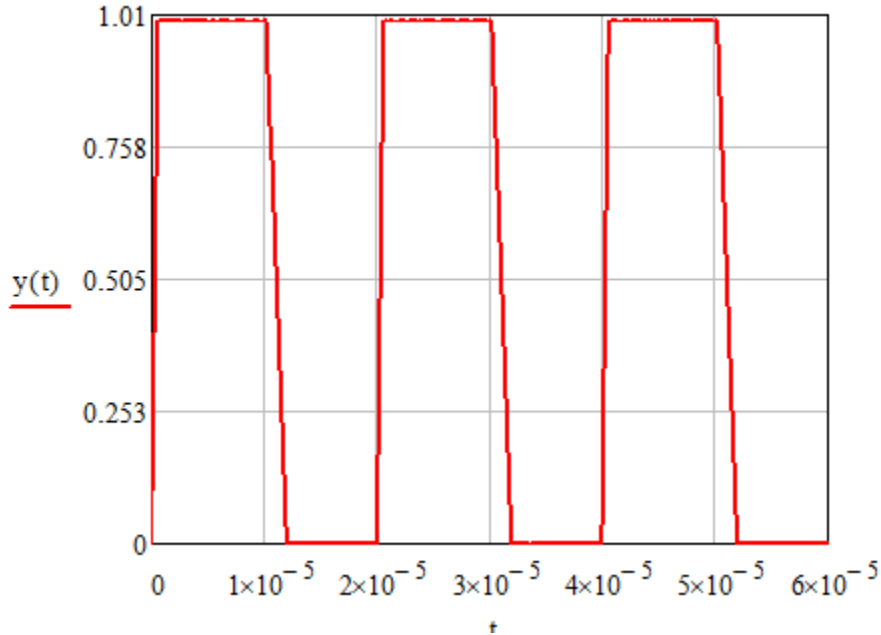


Fig 2. This graph is a proof of the validity of expression (1).

Therefore, we have justified the validity of expression (1).

Now, we can apply the Laplace transform to equation (1) to further use it for defining the resulting expression for a pulse train that went through, say, a low pass filter. Hence, the Laplace image of the original per (1) is

$$L_y(s) = \frac{(t_{off}-t_{off} \cdot e^{-s \cdot t_{on}} - t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} + t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} \cdot e^{-s \cdot t_{off}}) \cdot (e^{-T_s \cdot s} + e^{-2 \cdot T_s \cdot s} + e^{-3 \cdot T_s \cdot s} + 1)}{s^2 \cdot t_{on} \cdot t_{off}} \quad (2)$$

Now, define the transfer function (Laplace transform) for a simple low-pass filter as

$$F_{lp}(s) = \frac{1}{\tau_f \cdot s + 1} \quad (3)$$

where τ_f is the filter time constant.

Therefore, the Laplace image of the pulse train at the output of the low-pass filter will be

$$L_y(s) = L_y(s) \cdot F_{lp}(s) \quad (4)$$

or

$$L_f(s) = \frac{(t_{off}-t_{off} \cdot e^{-s \cdot t_{on}} - t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} + t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} \cdot e^{-s \cdot t_{off}}) \cdot (e^{-T_s \cdot s} + e^{-2 \cdot T_s \cdot s} + e^{-3 \cdot T_s \cdot s} + 1)}{s^2 \cdot t_{on} \cdot t_{off}} \cdot \frac{1}{\tau_f \cdot s + 1} \quad (5)$$

Now, we can go back to the time domain by applying the inverse Laplace transform to (5):

$$L_f(t) = t_{on} \cdot \left(\Phi(2 \cdot T_s - t + t_p + t_{on}) - 1 \right) \cdot \left(2 \cdot T_s - t + t_p + t_{on} + \tau_f - \tau_f \cdot e^{\frac{2 \cdot T_s - t + t_p + t_{on}}{\tau_f}} \right) - t_{off} \cdot \left(t - \tau_f + \tau_f \cdot e^{-\frac{t}{\tau_f}} \right) + t_{on} \cdot \left(\Phi(3 \cdot T_s - t + t_p + t_{on}) - 1 \right) \cdot \left(3 \cdot T_s - t + t_p + t_{on} + \tau_f - \tau_f \cdot e^{\frac{3 \cdot T_s - t + t_p + t_{on}}{\tau_f}} \right) \quad (6)$$

Example 1: Obtaining The Expression And Waveform For The Filtered Pulse Train

Plug in the real parameters listed here into equations (1) and (6):

$T_s = 20 \mu s$

$t_{on} = 0.6 \mu s$

$t_p = 9.6 \mu s$

$t_{off} = 1.8 \mu s$

$\tau_f = 1 \mu s$

Then, plot the initial pulse train (before the filter) and the pulse train at the output of the filter (using the following expression with $n_{max} = 3$) on the same graph as shown in Fig. 3:

$$y(t) = \sum_{n=0}^{n_{max}} \left[\Phi \left[(t - n \cdot T_s) \cdot \frac{t - n \cdot T_s}{t_{on}} - \frac{(t - t_{on} - n \cdot T_s)}{t_{on}} \cdot \Phi(t - t_{on} - n \cdot T_s) \right] - \left(\frac{t - t_{on} - t_p - n \cdot T_s}{t_{off}} \right) \cdot \Phi(t - t_{on} - t_p - n \cdot T_s) + \frac{t - t_{on} - t_p - n \cdot T_s - t_{off}}{t_{off}} \cdot \Phi(t - t_{on} - t_p - n \cdot T_s - t_{off}) \right]$$

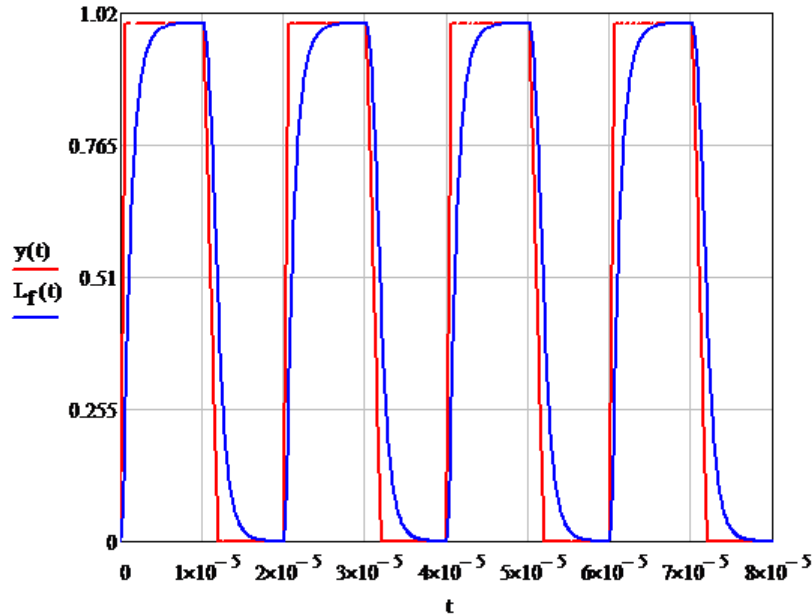


Fig. 3. An analytic expression for a pulse train of arbitrary shape allows for applying basic operations to the pulse train, obtaining plausible results at the output. The filter defines the rise and fall times only.

Example 2. Obtaining A Filtered Pulse Train At A Higher Filter Time Constant

In this case, let's apply the same parameters as in example 1, but with $\tau_f = 20 \mu s$. The results for the two pulse trains are shown in Fig. 4.

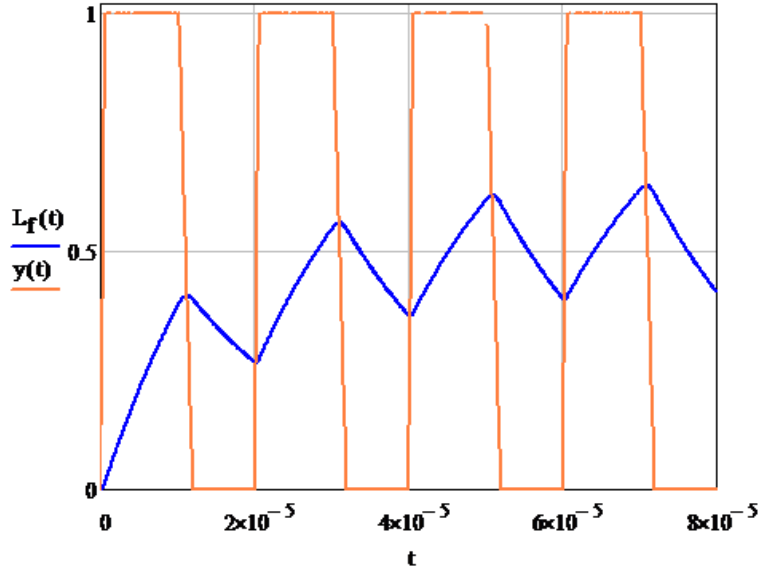


Fig. 4. The filter defines the whole output process slew rate.

Example 3: Pulse Trains Applied To Half-Bridge-Configured MOSFETs

The gate-source signals at the half-bridge MOSFETs should be shifted by $T_s/2$. Thus, one of the signals may be

$$y(t) \text{ and another one will be } y\left(t - \frac{T_s}{2}\right).$$

These signals based on equation (1) and using the timing parameters from example 1 are shown in Fig. 5.

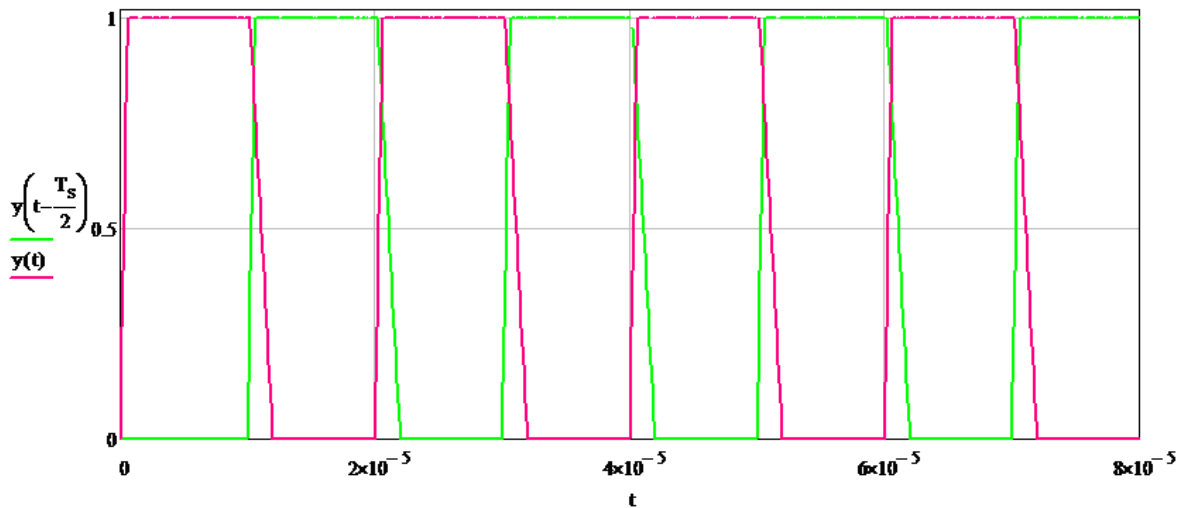


Fig. 5. The pulses per Fig. 2 cannot be used for driving a half-bridge due to overlapping.

Based on the Fig. 5 legend we conclude that the pulse train pulses' rise and fall times should be adjusted to avoid overlapping when driving half-bridge MOSFETs. We can accomplish this by adjusting the pulse parameters as follows:

$$t_{on} = 0.14 \mu\text{s}$$

$$t_p = 9.1 \mu\text{s}$$

$$t_{off} = 0.18 \mu\text{s}$$

Then, applying these values to the pulse train expression

$$y(t) = \sum_{n=0}^{n_{\max}} \left[\Phi \left[(t - n \cdot T_s) \cdot \frac{t - n \cdot T_s}{t_{on}} - \frac{(t - t_{on} - n \cdot T_s)}{t_{on}} \cdot \Phi(t - t_{on} - n \cdot T_s) \right] \right. \\ \left. - \left(\frac{t - t_{on} - t_p - n \cdot T_s}{t_{off}} \right) \cdot \Phi(t - t_{on} - t_p - n \cdot T_s) + \frac{t - t_{on} - t_p - n \cdot T_s - t_{off}}{t_{off}} \cdot \Phi(t - t_{on} - t_p - n \cdot T_s - t_{off}) \right]$$

we obtain the waveforms shown in Fig. 6.

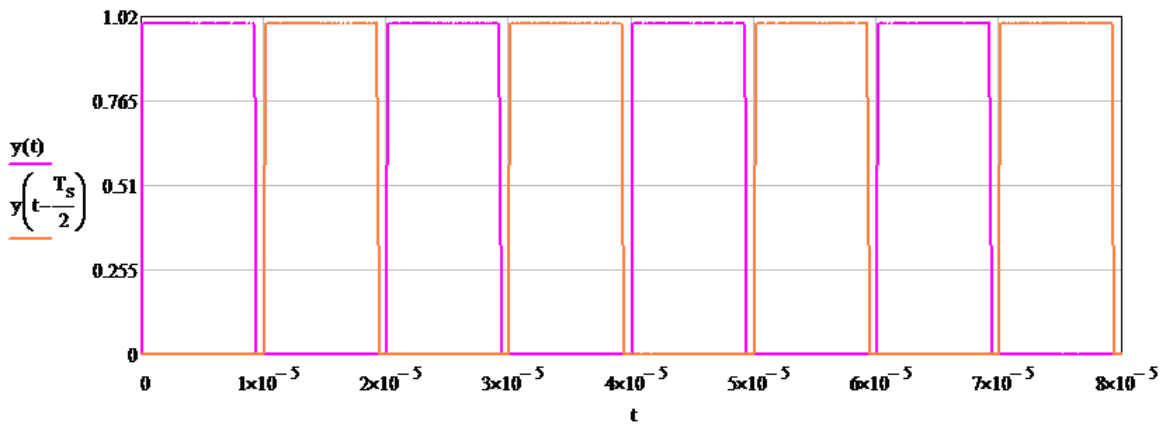


Fig. 6. After adjustment of the rise and fall times these pulses can drive half-bridge MOSFETs.

In the above example, the application of the Laplace transform was not needed. It was sufficient simply to adjust some parameters in the expression for the pulse train, $y(t)$, reflecting the changes in circuit parameters. However, if filtering was applied to the circuit and distorted the waveforms as in Fig. 3, a Laplace transform would immediately reveal what filter parameters would need to be tweaked.

Takeaway

An analytic expression for a series of trapezoidal pulses provides a versatile means of analysis and simulation of diverse parameters of switching power supplies. This expression may be a basis for the SMPS simulation and building new SMPS units.

References

1. ["Using Duty Cycle To Detect Power Supply Overload \(Part 2\): A Practical Implementation"](#) by Gregory Mirsky, How2Power Today, May 2017 issue.

2. "[Switching Power Supplies And EMI: Debunking The Myths About Frequency And Slew Rate](#)" by Gregory Mirsky, How2Power Today, April 2018 issue.

About The Author



Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an

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Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels [online](#).

For more on the analysis of deadtime in switching waveforms, see How2Power's [Design Guide](#) and do a key word search on the terms "deadtime" and "dead time".