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A Guide To Designing Your Own Rogowski Sensor (Part 1)

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Current measurement is a common requirement in many power converter and inverter designs. Traction inverters are just one example. Rogowski sensors are an interesting option in this case as they have a very wide current range, very high accuracy (up to 0.1%), and are non-saturating. On the down side, they are very expensive, costing as much as $3000 for an instrument-grade sensor.

However, in cases such as a traction inverter, the full accuracy of an off-the shelf Rogowski coil sensor may not be needed. In this case, it may be possible to design and build a lower-cost sensor that retains most of the benefits of this current-measuring sensor. To that end, I have developed, calculated and simulated a prototype of a Rogowski current sensor that can be much less expensive than the existing ones but still have very good accuracy.

Implementing this sensor design depends mainly on an understanding of Rogowski coil operation and having the key formulas available for designing the coil and the integrator circuit. The formulas defining the operation of the Rogowski coil will be derived here in part 1. Then in part 2, details of the integrator design will be explained. Finally, in part 3, I'll present simulations of the final sensor design.

Defining Key Terminology

Before discussing any details of sensor design, let's establish the terminology that needs to be understood. The Rogowski term can be used variously to describe a coil, a probe and a sensor.

A Rogowski probe is the complete instrument composed of the coil and integrator, and is designated to operate as a standalone instrument. A Rogowski sensor is the complete instrument, similar to the probe, but is intended to work in a system as a part of the system, continuously. Meanwhile, a Rogowski coil is just the wire-wound coil around a dielectric flexible core, not connected to any integrator.

Rogowski ac current probes (sensors) are very accurate versatile and convenient instruments that can be used for measuring ac currents over very wide ranges of values and frequencies. They may be a part of current measuring instruments as well as standalone current sensors operating as a part of a control loop.

Rogowski sensors are based on a non-magnetic current measuring method, which does not employ any magnetic materials and therefore these sensors never saturate. They have a "disconnectable" sensing coil that can be wrapped around the current-carrying conductor or bus and locked to itself, providing access to wires, pins and buses concealed in the depth of electronic and electric equipment. The sensing coil is terminated by an integrator. A concise Rogowski probe schematic layout is depicted in Fig. 1.

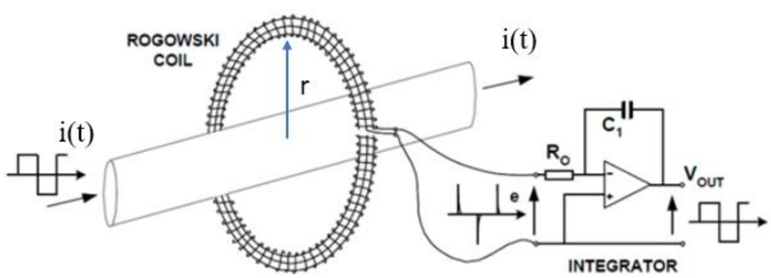


Fig. 1. Schematic of the Rogowski probe. The Rogowski coil wires are connected to an integrator that compensates for the differentiating property of the mutual inductance between the bus carrying current i(t) and the coil placed at a distance r from the bus. Diagram courtesy of Keysight (r and i(t) added here) (see the reference).

Determining The Rogowski Coil's Timing Response

First of all, let's consider the simplified schematic diagram for a Rogowski coil and find its timing response to a single trapezoidal pulse. We are working with trapezoidal pulses since they better reflect real pulses which occur in electronic devices, and they have a wide spectrum. If the probe reproduces a trapezoidal pulse correctly, it will reproduce a harmonic oscillation correctly, too.

For simplicity of the Laplace transform equations that we will be using in this analysis, we will consider the action of a single pulse only. Subsequent results will justify that.

Let's consider Fig. 2 depicting the Rogowski coil parameters:

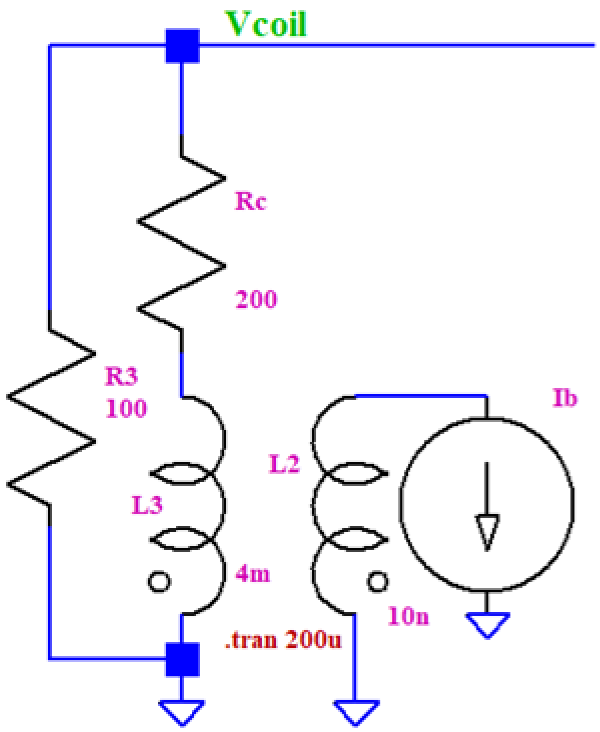


Fig. 2. This schematic diagram of a Rogowski coil also has a part showing the bus (current carrying conductor) represented by inductance L2 and the current I_b to measure. Coupling between bus L2 and coil L3 is weak since there is a big air gap between them. Rc is the coil's intrinsic resistance and R3 is a low-value terminating resistor, which reduces noise and stabilizes the output shape of the Vcoil waveform.

For convenience, let's define some of the crucial components of the equations we'll use in this analysis:

$I_b(t)$ = the bus current to measure

N_b = number of the current-carrying bus turns

l_c = length of the Rogowski coil

μ_r = relative permeability of the coil core. In our case $\mu_r = 1$.

μ_0 = absolute permeability of free space

$\mu = \mu_0 \cdot \mu_r$

$H(t)$ = magnetic field strength created by current in the bus $I_b(t)$

$B(t)$ = magnetic flux density that crosses the coil turns

$\psi(t)$ = magnetic flux linkage that crosses the coil turns

S_c = coil cross-sectional area

L2 = inductance of the coil loop that embraces the bus

L3 = inductance of the coil formed by the winding

R_c = intrinsic resistance of the Rogowski coil L3

M = mutual inductance between L2 and L3

R3 = termination resistance of the Rogowski coil

V_{coil} = voltage between the wires of the Rogowski sensor coil

V_{L3} = Voltage across L3 induced by the magnetic field produced by the bus.

We now have to determine how the Rogowski coil reacts to a trapezoidal pulse of the measured current. Using Ampere's law (the Full Current Law), we can define the magnetic flux linkage in the coil:

Ampere's law:

$$H(t) \cdot l_c = I_b(t) \cdot N_b \qquad (1)$$

Hence the magnetic field crossing the coil turns is

$$H(t) = \frac{I_b(t) \cdot N_b}{l_c} \qquad (2)$$

Therefore, flux density

$$B(t) = H(t) \cdot \mu = \mu \cdot \frac{I_b(t) \cdot N_b}{l_c} \qquad (3)$$

and linkage flux

$$\psi(t) = S_c \cdot N_w \cdot B(t) = S_c \cdot N_w \cdot \left(\mu \cdot \frac{I_b(t) \cdot N_b}{l_c}\right) \qquad (4)$$

Designating the mutual inductance as M, we obtain the voltage generated across the wires of the Rogowski coil regardless of R_c as:

$$V_{L3}(t) = \left(\frac{d}{dt}\psi(t)\right) = S_c \cdot N_w \cdot N_b \cdot \frac{\mu}{l_c} \cdot \left(\frac{d}{dt} I_b(t)\right) = M \cdot \frac{d}{dt} I_b(t) \qquad (5)$$

Applying the Laplace transform, we get

$$V_{L3}(t) = s \cdot M \cdot I_b(s) \qquad (6)$$

which yields the transfer function that has a dimension of inductance M in the time domain.

$$M = S_c \cdot N_w \cdot N_b \cdot \frac{\mu}{l_c}$$

Using (6) we can define the Rogowski coil transfer function as

$$G_{coil}(s) = \frac{V_{coil}(s)}{I_b(s)} \qquad (7)$$

This gives us the basis for defining V_{coil} produced by the Rogowski coil. But we would like to express this voltage as a function of I_b as a trapezoidal current pulse and with regard to the equivalent schematic per Fig. 2.

First, we observe that a single bus current pulse can be described this way:

$$I_b(t) = I_{b0} \cdot \left[\left[\Phi(t) \cdot \frac{t}{t_{on}} - \frac{(t-t_{on})}{t_{on}} \cdot \Phi(t-t_{on})\right] - \left[\frac{(t-t_{on}-t_p)}{t_{off}}\right] \cdot \Phi(t-t_{on}-t_p) + \frac{t-t_{on}-t_p-t_{off}}{t_{off}} \cdot \Phi(t-t_{on}-t_p-t_{off})\right] \qquad (8)$$

Taking the Laplace transform of the above expression, simplifying and assuming $t_{on} > 0$, $t_p > 0$ and $t_{off} > 0$, we obtain

$$\frac{I_{b0} \cdot (t_{off} - t_{off} e^{-s \cdot t_{on}} - t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} + t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} \cdot e^{-s \cdot t_{off}})}{s^2 \cdot t_{on} \cdot t_{off}} \qquad (9)$$

Using (6) again we get:

$$V_{L3}(s) = M \cdot \frac{I_{b0} \cdot (t_{off} - t_{off} e^{-s \cdot t_{on}} - t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} + t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} \cdot e^{-s \cdot t_{off}})}{s^2 \cdot t_{on} \cdot t_{off}} \qquad (10)$$

How are $V_{L3}(s)$ and $V_{coil}(s)$ related? Let's refer back to Rogowski coil schematic from Fig. 2, which is repeated here in Fig. 3 for the reader's convenience.

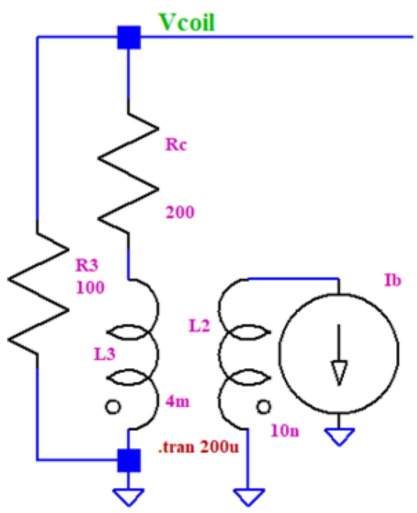


Fig. 3. Rogowski coil schematic.

From Fig. 3 we see:

$$V_{coil}(s) = \frac{V_{L3}(s)}{s \cdot L3 + R_c + R3} \cdot R3 = V_{L3}(s) \cdot \frac{R3}{(R_c + R3) \cdot \left(1 + s \cdot \frac{L3}{R_c + R3}\right)} = V_{L3}(s) \cdot \frac{\sigma_0}{1 + s \cdot \tau_0} \qquad (11)$$

Here

$$\sigma_0 = \frac{R3}{R3+R_c} \quad (12)$$

$$\tau_0 = \frac{L3}{R3+R_c} \quad (13)$$

Plugging equation (10) into (11), we obtain:

$$V_{coil}(s) = \frac{\sigma_0}{(1+s\cdot\tau_0)} \cdot \left[M \cdot \frac{I_{b0}\cdot(t_{off} - t_{off}\cdot e^{-s\cdot t_{on}} - t_{on}\cdot e^{-s\cdot t_p}\cdot e^{-s\cdot t_{on}} + t_{on}\cdot e^{-s\cdot t_p}\cdot e^{-s\cdot t_{on}}\cdot e^{-s\cdot t_{off}})}{s\cdot t_{on}\cdot t_{off}}\right] \quad (14)$$

Taking the inverse Laplace of the above expression, simplifying and assuming $s > 0$, $t_{on} > 0$, $t_{off} > 0$ and $\tau_0 > 0$, yields

$$I_{b0} \cdot M \cdot \sigma_0 \left(t_{off} - t_{off} \cdot e^{-\frac{t}{\tau_0}} + t_{on} \cdot \Phi(t - t_p - t_{on} - t_{off}) - t_{on} \cdot \Phi(t - t_p - t_{on}) - t_{off} \cdot \Phi(t - t_{on}) + t_{off} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0}} \cdot \Phi(t - t_{on}) + t_{on} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_p}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0 \cdot}} \cdot \Phi(t - t_p - t_{on}) - t_{on} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_p}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0 \cdot}} \cdot e^{\frac{t_{off}}{\tau_0 \cdot}} \cdot \Phi(t - t_p - t_{on} - t_{off}) \right) \cdot \frac{1}{t_{on} \cdot t_{off}}$$

which yields:

$$V_{coil}(t) = M \cdot \sigma_0 \left(t_{off} - t_{off} \cdot e^{-\frac{t}{\tau_0}} + t_{on} \cdot \Phi(t - t_p - t_{on} - t_{off}) - t_{on} \cdot \Phi(t - t_p - t_{on}) - t_{off} \cdot \Phi(t - t_{on}) + t_{off} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0}} \cdot \Phi(t - t_{on}) + t_{on} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_p}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0 \cdot}} \cdot \Phi(t - t_p - t_{on}) - t_{on} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_p}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0 \cdot}} \cdot e^{\frac{t_{off}}{\tau_0 \cdot}} \cdot \Phi(t - t_p - t_{on} - t_{off}) \right) \cdot \frac{1}{t_{on}\cdot t_{off}} \quad (15)$$

To illustrate the application of the above equations, let's now define parameters of a realistic Rogowski coil and plug them in into the calculations below:

Set:

$N_b = 1$

$\mu = \mu_0$

$l_c = 100$ mm

R3 = 10.0 Ω

Wire diameter: $d_W = 0.05$ mm

Number of turns: $N_w = \frac{l_c}{d_W} = 2 \times 10^3$

One turn diameter: $d_{turn} = 1.5$ mm

Coil wire one-turn length: $l_{turn} = \pi \cdot d_{turn} = 4.712 \text{ x } 10^{-3} \text{m}$

Coil wire length: $l_W = l_{turn} \cdot N_w = 9.425 \text{ m}$

Coil winding one turn area: $S_C = \pi \cdot \frac{d_{turn}^2}{4} = 1.767 \text{ x } 10^{-6} \text{m}^2$

Coil inductance: $L3 = N_W^2 \cdot \frac{S_C}{l_c} \cdot \mu_0 = 88.826 \text{ x } 10^{-6} \text{H}$

Coil resistance:

$\rho_{CU} = 1.68 \text{ x } 10^{-8} \Omega \cdot \text{m}$

$$R_C = \rho_{CU} \cdot \frac{l_W}{\pi \cdot \frac{d_W^2}{4}} = 80.64\ \Omega$$

Mutual inductance

$$M = S_C \cdot N_W \cdot N_b \cdot \frac{\mu}{l_c} = 44.413 \text{ x } 10^{-9} \text{H}$$

Also, for demonstration purposes, let’s plug in real values of variables and constants:

$M = 4.441 \text{ x } 10^{-8}$ H

$R_c = 80.64\ \Omega$

$L3 = 8.883 \text{ x } 10^{-5}$ H

$S_c = 1.767 \text{ x } 10^{-6} \text{ m}^2$

$l_W = 9.425$ m

$l_{turn} = 4.712 \text{ x } 10^{-3}$ m

$d_{turn} = 1.5 \text{ x } 10^{-3}$ m

$N_W = 2 \text{ x } 10^3$

$d_W = 5 \text{ x } 10^{-5}$ m

$t_{on} = 40 \text{ x } 10^{-9}$ s

$t_p = 50$ µs

$t_{off} = 60 \text{ x } 10^{-9}$ s

$$\sigma_0 = \frac{R3}{R3 + R_c} = 0.11$$

$$\tau_0 = \frac{L3}{R3 + R_c} = 9.8\ x\ 10^{-7} \text{s}$$

$I_{b0} = 200$ A

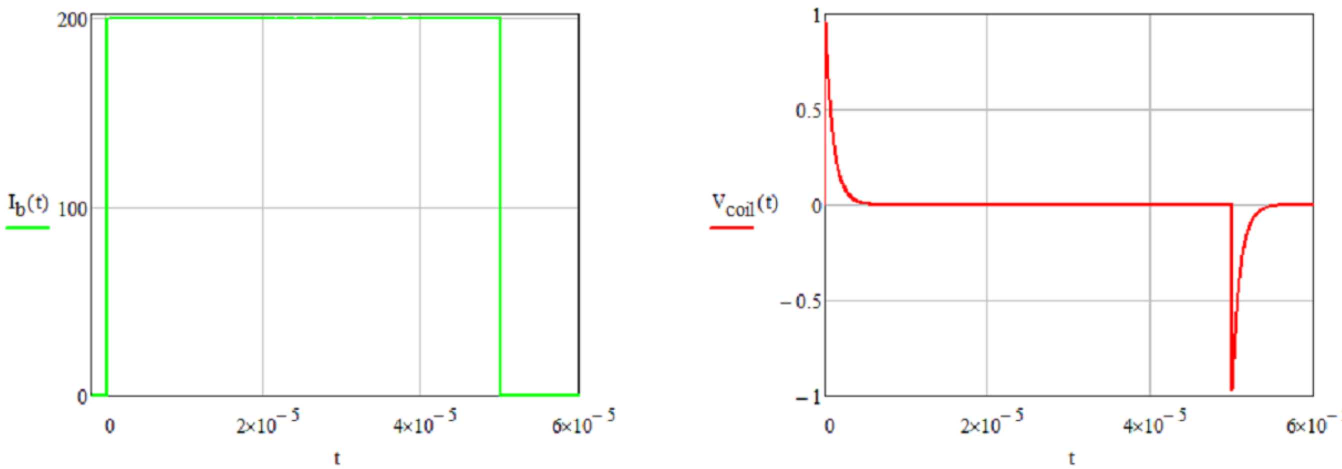
$R3 = 10\ \Omega$

Finally, let's recall equations (8) and (15) that were derived above:

$$I_b(t) = I_{b0} \cdot \left[\left[\Phi(t) \cdot \frac{t}{t_{on}} - \frac{(t - t_{on})}{t_{on}} \cdot \Phi(t - t_{on}) \right] - \left[\frac{(t - t_{on} - t_p)}{t_{off}} \right] \cdot \Phi(t - t_{on} - t_p) + \frac{t - t_{on} - t_p - t_{off}}{t_{off}} \cdot \Phi(t - t_{on} - t_p - t_{off}) \right]$$

$$V_{coil}(t) = M \cdot \sigma_0 \left(t_{off} - t_{off} \cdot e^{-\frac{t}{\tau_0}} + t_{on} \cdot \Phi(t - t_p - t_{on} - t_{off}) - t_{on} \cdot \Phi(t - t_p - t_{on}) - t_{off} \cdot \Phi(t - t_{on}) + t_{off} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0}} \cdot \Phi(t - t_{on}) + t_{on} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_p}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0}} \cdot \Phi(t - t_p - t_{on}) - t_{on} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_p}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0}} \cdot e^{\frac{t_{off}}{\tau_0}} \cdot \Phi(t - t_p - t_{on} - t_{off}) \right) \cdot \frac{1}{t_{on} \cdot t_{off}}$$

If we plug in the values assigned above into these equations, we can plot $I_b(t)$ and $V_{coil}(t)$ as shown in Fig. 4.



(a) *(b)*

Fig. 4. Demonstration of the trapezoidal current waveform to measure $I_b(t)$ (a) and voltage generated by the Rogowski coil $V_{coil}(t)$ (b).

From these plots, we see that a rectangular current waveform $I_b(t)$ in Fig. 3a turns into an exponential waveform for $V_{coil}(t)$ in Fig. 3(b). Therefore, we have to integrate the waveform for $V_{coil}(t)$ to restore its rectangular waveshape to obtain an authentic representation of the bus current $I_b(t)$ waveform.

So, we have to push the exponential pulse through the integrator shown in Fig. 5 to obtain the rectangular output.

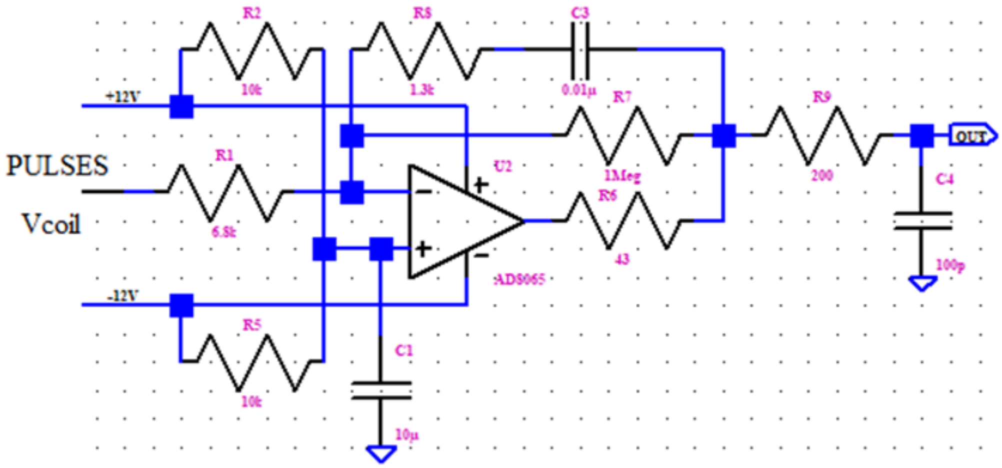


Fig. 5. An integrator based on a fast operational amplifier can restore the initial measured current waveform and scale it to the necessary output voltage range. C3, R7 and R1 are integrating components. R8 is a correction resistor that allows for an accurate tweaking of the output voltage waveform.

In part 2, we will review how to design the integrator in order to correct $V_{coil}(t)$ so that it accurately reproduces the bus current waveform.

Reference

"What is a Rogowski Coil Current Probe?" Keysight website, accessed April 5, 2024.

About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

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