ISSUE: May 2024

A Guide To Designing Your Own Rogowski Sensor (Part 2)

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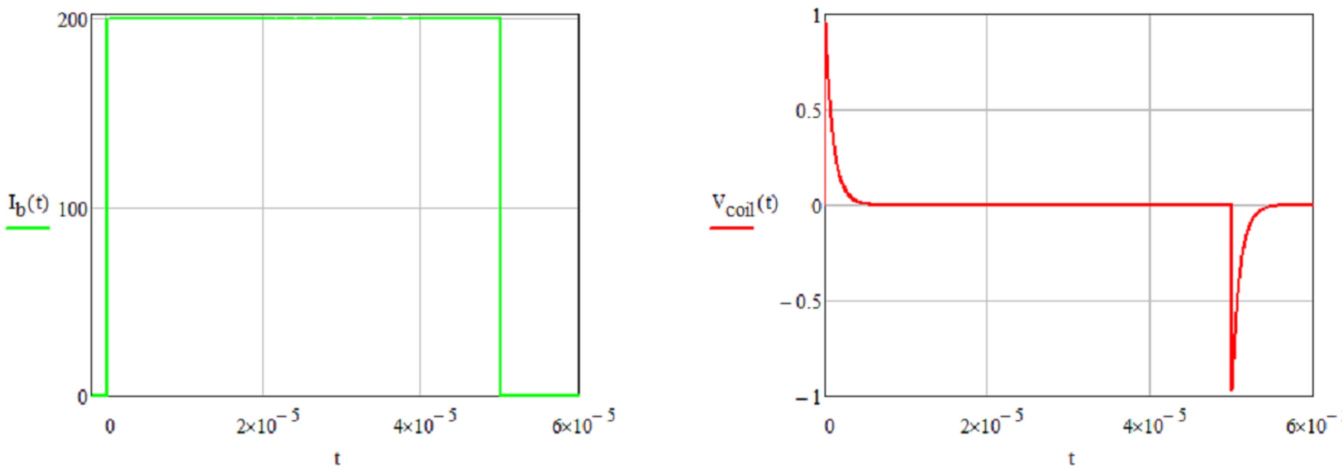
In part 1[1] we learned that a response to the trapezoidal current of a Rogowski coil (Fig. 1a) is the voltage waveform shown in Fig. 1b, which is essentially a derivative of the measured current waveform. To restore the current waveform, we have to integrate the coil's output voltage.



(a) *(b)*

Fig. 1. Demonstration of the trapezoidal current waveform to measure $I_b(t)$ (a) and voltage generated by the Rogowski coil $V_{coil}(t)$ (b).

Analytical expressions for these waveforms, which were derived in part 1, are repeated below for the reader's convenience.

$$I_b(t) = I_{b0} \cdot \left[\left[\Phi(t) \cdot \frac{t}{t_{on}} - \frac{(t - t_{on})}{t_{on}} \cdot \Phi(t - t_{on})\right] - \left[\frac{(t - t_{on} - t_p)}{t_{off}}\right] \cdot \Phi(t - t_{on} - t_p) + \frac{t - t_{on} - t_p - t_{off}}{t_{off}} \cdot \Phi(t - t_{on} - t_p - t_{off})\right]$$

$$V_{coil}(t) = I_{b0} \cdot M \cdot \sigma_0 \left(t_{off} - t_{off} \cdot e^{-\frac{t}{\tau_0}} + t_{on} \cdot \Phi(t - t_p - t_{on} - t_{off}) - t_{on} \cdot \Phi(t - t_p - t_{on}) - t_{off} \cdot \Phi(t - t_{on}) + t_{off} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0}} \cdot \Phi(t - t_{on}) + t_{on} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_p}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0}} \cdot \Phi(t - t_p - t_{on}) - t_{on} \cdot e^{-\frac{t}{\tau_0}} \cdot e^{\frac{t_p}{\tau_0}} \cdot e^{\frac{t_{on}}{\tau_0}} \cdot e^{\frac{t_{off}}{\tau_0}} \cdot \Phi(t - t_p - t_{on} - t_{off})\right) \cdot \frac{1}{t_{on} \cdot t_{off}}$$

So, as noted in part 1, we have to push the exponential pulse through the integrator shown in Fig. 2 to obtain the rectangular output. In this second part of the article, we'll derive the integrator's transfer function and then show how this is used to select the integrator's key components.

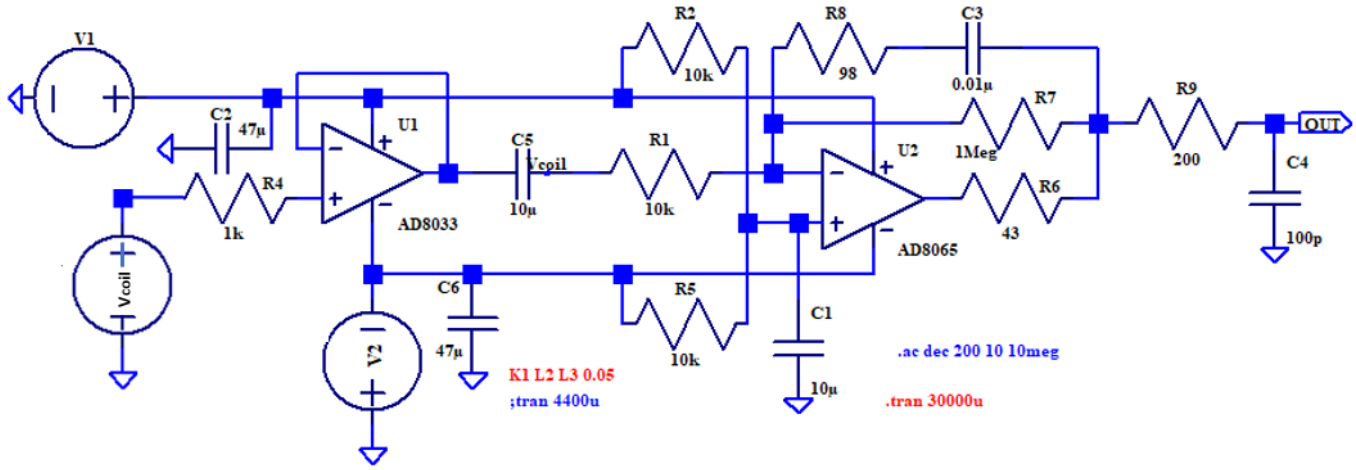


Fig. 2. An integrator based on a fast operational amplifier U2 can restore the initial measured current waveform and scale it to the necessary output voltage range. C3, R7 and R1 are integrating components. R8 is a correction resistor that allows for an accurate tweaking of the output voltage waveform. The selection of the component values shown here will be discussed in the design example that follows.

Integrator Design

First of all, let's define the integrator's transfer function. In Fig. 2, capacitor C3 and resistor R8 make up the correction circuit that is parallel to the feedback resistor R7. Designate the correction circuit R8-C3 impedance as

$$Z_{corr}(s) = R8 + \frac{1}{s \cdot C3}$$

or

$$Z_{corr}(s) = \frac{C3 \cdot R8 \cdot s + 1}{C3 \cdot s} \qquad (1)$$

The feedback circuit impedance is composed of R7 and $Z_{corr}(s)$ where R7 and $Z_{corr}(s)$ are connected in parallel to make up

$$Z_{FB}(s) = \frac{R7 \cdot \frac{C3 \cdot R8 \cdot s + 1}{C3 \cdot s}}{R7 + \frac{C3 \cdot R8 \cdot s + 1}{C3 \cdot s}}$$

$$Z_{FB}(s) = \frac{R7 \cdot (C3 \cdot R8 \cdot s + 1)}{(C3 \cdot R7 + C3 \cdot R8) \cdot s + 1} \qquad (2)$$

The integrator transfer function

$$G_{int}(s) = \frac{Z_{FB}(s)}{R1} = \frac{\frac{R7 \cdot (C3 \cdot R8 \cdot s + 1)}{(C3 \cdot R7 + C3 \cdot R8) \cdot s + 1}}{R1}$$

which reduces to

$$G_{int}(s) = \frac{R7 \cdot (C3 \cdot R8 \cdot s + 1)}{R1 \cdot [(C3 \cdot R7 + C3 \cdot R8) \cdot s + 1]} \tag{3}$$

For simplicity, we'll designate some time-constants and a resistor ratio:

$\tau_{38} = C3 \bullet R8$

$\tau_{37} = C3 \bullet R7$

$\alpha = R7/R1$

$\tau_{37} + \tau_{38} = \tau_{378}$

This allows the transfer function to be rewritten as

$$G_{int}(s) = \alpha \cdot \frac{(\tau_{38} \cdot s + 1)}{\tau_{378} \cdot s + 1} \tag{4}$$

Therefore, the output voltage V_{OUT} can be described in the frequency domain as

$$V_{out}(s) = V_{coil}(s) \cdot G_{int}(s)$$

Substituting the expression for $V_{coil}(s)$ from above yields

$$V_{out}(s) = \frac{\sigma_0}{(1 + s \cdot \tau_0)} \cdot \left[M \cdot \frac{I_{b0} \cdot (t_{off} - t_{off} \cdot e^{-s \cdot t_{on}} - t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} + t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} \cdot e^{-s \cdot t_{off}})}{s \cdot t_{on} \cdot t_{off}} \right] \cdot \left[\alpha \cdot \frac{(\tau_{38} \cdot s + 1)}{\tau_{378} \cdot s + 1} \right]$$

$$V_{out}(s) = \sigma_0 \cdot (M \cdot I_{b0}) \cdot \alpha \cdot \left[\frac{(\tau_{38} \cdot s + 1)}{(\tau_{378} \cdot s + 1) \cdot (1 + s \cdot \tau_0)} \right] \cdot \frac{(t_{off} - t_{off} \cdot e^{-s \cdot t_{on}} - t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} + t_{on} \cdot e^{-s \cdot t_p} \cdot e^{-s \cdot t_{on}} \cdot e^{-s \cdot t_{off}})}{s \cdot t_{on} \cdot t_{off}} \tag{5}$$

Now, it is time to go to the time-domain and after the inverse Laplace transform obtain:

$$V_{out}(t) = I_{b0} \cdot M \cdot \alpha \cdot \sigma_0 \cdot \begin{bmatrix} e^{\frac{t_p - t + t_{on} + t_{off}}{\tau_0}} \cdot \left(t_{on} \cdot \tau_0 - t_{on} \cdot \tau_{38} + t_{on} \cdot \tau_{38} \cdot \Phi\left(t_p - t + t_{on} + t_{off}\right) \right) \\ -e^{\frac{t_p - t + t_{on} + t_{off}}{\tau_{378}}} \cdot \left(t_{on} \cdot \tau_{378} - t_{on} \cdot \tau_{38} + t_{on} \cdot \tau_{38} \cdot \Phi\left(t_p - t + t_{on} + t_{off}\right) \right) \\ -e^{\frac{t_p - t + t_{on}}{\tau_0}} \cdot \left(t_{on} \cdot \tau_0 - t_{on} \cdot \tau_{38} + t_{on} \cdot \tau_{38} \cdot \Phi\left(t_p - t + t_{on}\right) \right) + e^{\frac{t_p - t + t_{on}}{\tau_{378}}} \end{bmatrix} \tag{6}$$

To demonstrate the use of the above expression for $V_{out}(t)$, the author has calculated the following integrator component values based on the required frequency and phase characteristics of the integrator. As these are very lengthy calculations, only the results are presented here:

C3 = 0.01 µF

R7 = 1 MΩ

R8 = 98 Ω

R1 = 10 kΩ

Then, we also require the following physical values of components, which were calculated by the author for a Rogowski sensor used in an EV inverter. (Some of the calculations were shown in part 1.)

I_{b0} = 600 A

$M = 44 \bullet 10^{-9}$ H

R3 = 10 Ω

R_c = 80.64 Ω

L3 = 89 μH

$\tau_0 = L3/(R3 + R_c) = 0.98 \times 10^{-6}$ s

t_{on} = 40 ns

t_p = 50 μs

t_{off} = 60 ns

$\alpha = R7/R1 = 100$

$\sigma_0 = 0.11$

$\tau_{38} = C3 \bullet R8 = 0.98 \times 10^{-6}$ s

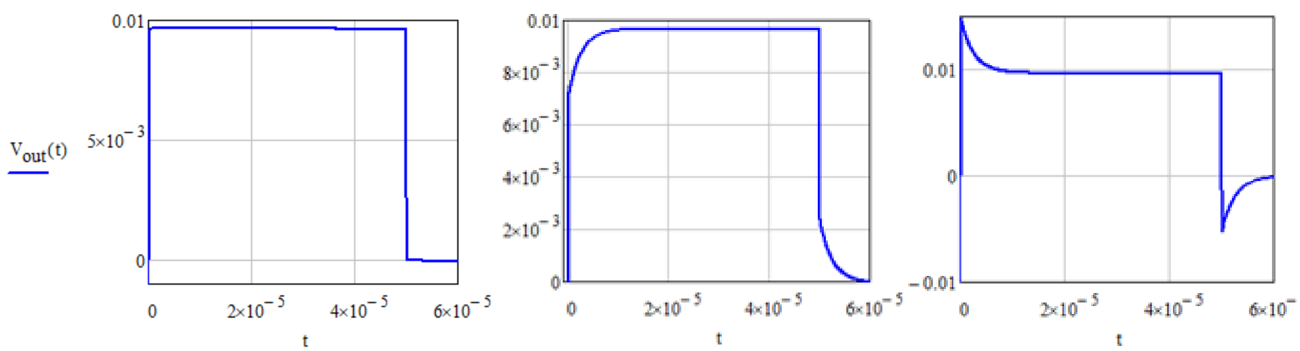
$\tau_{37} = C3 \bullet R7 = 0.01$ s

$\tau_{378} = \tau_{37} + \tau_{38} = 0.01$ s

Plugging the above values into the expression from above for $V_{out}(t)$:

$$V_{out}(t) = I_{b0} \cdot M \cdot \alpha \cdot \sigma_0 \cdot \begin{bmatrix} e^{\frac{t_p - t + t_{on} + t_{off}}{\tau_0}} \cdot \left(t_{on} \cdot \tau_0 - t_{on} \cdot \tau_{38} + t_{on} \cdot \tau_{38} \cdot \Phi\left(t_p - t + t_{on} + t_{off}\right)\right) \\ -e^{\frac{t_p - t + t_{on} + t_{off}}{\tau_{378}}} \cdot \left(t_{on} \cdot \tau_{378} - t_{on} \cdot \tau_{38} + t_{on} \cdot \tau_{38} \cdot \Phi\left(t_p - t + t_{on} + t_{off}\right)\right) \\ -e^{\frac{t_p - t + t_{on}}{\tau_0}} \cdot \left(t_{on} \cdot \tau_0 - t_{on} \cdot \tau_{38} + t_{on} \cdot \tau_{38} \cdot \Phi\left(t_p - t + t_{on}\right)\right) + e^{\frac{t_p - t + t_{on}}{\tau_{378}}} \end{bmatrix}$$

we can now plot the coil output voltage at different values of the correction resistor R8 as shown in Fig. 3.



a) R8 = 98 Ω   b) R8 = 88 Ω   c) R8 = 108 Ω

Fig. 3. The integrator output voltage at different resistance values of correction resistor R8. If using resistors having a tolerance of 0.5% or better and a capacitor C3 tolerance of 1%, no adjustment is required. Otherwise, the correction circuit needs adjustment.

Let's recall the schematic layout of the Fig. 2 integrator, repeated in Fig. 4 for the reader's convenience.

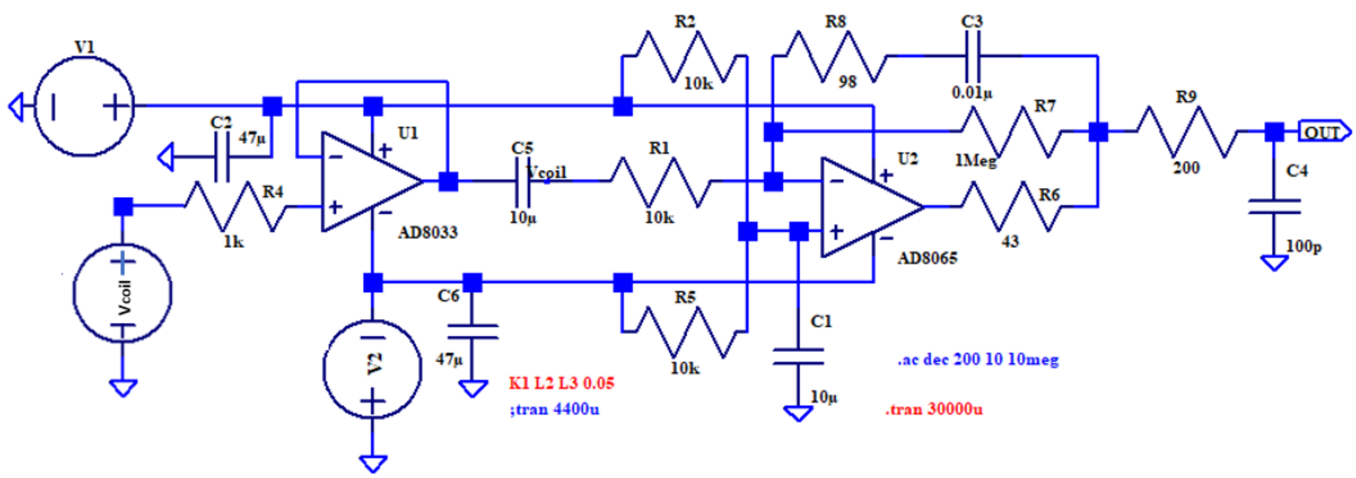


Fig. 4. An integrator based on a fast operational amplifier U2 can restore the initial measured current waveform and scale it to the necessary output voltage range.

Components R9 and C4 compose the output filter that snubs possible parasitic oscillations at the integrator's output. If we connect the output of the Rogowski coil in Fig. 5 to resistor R4 at the input of the integrator in Fig. 4, we obtain a usable schematic layout as shown below in Fig 6.

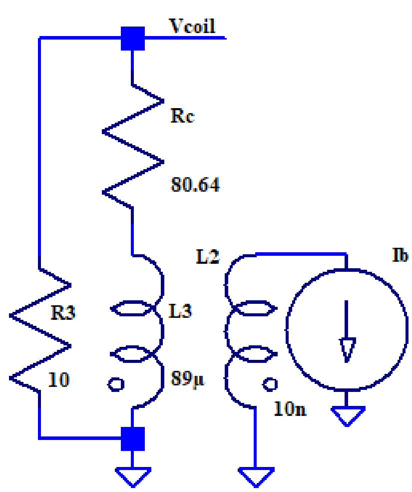


Fig. 5. Schematic of the Rogowski coil. Rc is the inductor L3 winding resistance.

Operational amplifiers should be stable at gain = -1 and have an operating frequency-gain product of 50 MHz to 100 MHz or more. The output of the shown sensor extends to a few tens of millivolts and can be increased if necessary by adding another amplifier in series through an adequate capacitor, ensuring the target gain within the frequency band of 10 Hz to 10 MHz or more.

This is the end of the discussion describing the integrator design. However, there are a few aspects of the design that merit further attention.

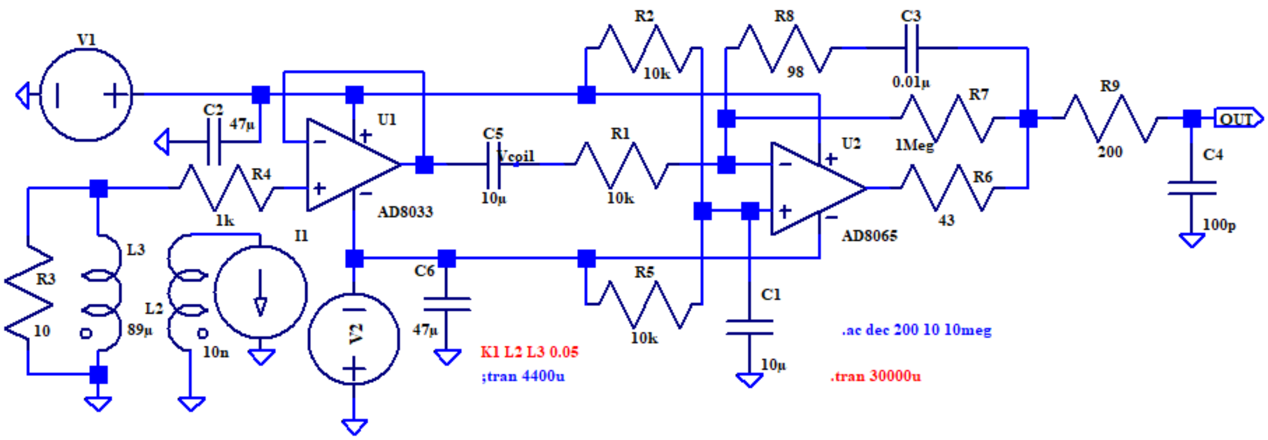


Fig. 6. LTSpice schematic layout of the Rogowski sensor that was simulated as a proof of the design. The integrator uses the AD8033 and AD8065 op amps.[2,3]

First, it is important is to keep the Rogowski sensor stable when two contradicting conditions are present: 1) a high gain and 2) wide enough operating frequency range, which is critical for accurately reproducing the waveform of the current under measurement. Secondly, it is also important to have all the electronic components stable, temperature-wise, and time-wise.

In some applications it is desirable to digitize current measurements. For such cases, it is feasible to use the Rogowski sensor with an analog-to-digital converter IC to further connect it to a CAN bus.

The next installment in this series, part 3, will be dedicated to the Rogowski sensor simulation results and their interpretation.

Reference

1. "A Guide To Designing Your Own Rogowski Sensor (Part 1)" by Gregory Mirsky, How2Power Today, April 2024.

2. AD8065/8066 datasheet.

3. AD8033/8034 datasheet.

About The Author

Gregory Mirsky is a design engineer working in Deer Park, Ill. He currently performs design verification on various projects, designs and implements new methods of electronic circuit analysis, and runs workshops on MathCAD 15 usage for circuit design and verification. He obtained a Ph.D. degree in physics and mathematics from the Moscow State Pedagogical University, Russia. During his graduate work, Gregory designed hardware for the high-resolution spectrometer for research of highly compensated semiconductors and high-temperature superconductors. He also holds an MS degree from the Baltic State Technical University, St. Petersburg, Russia where he majored in missile and aircraft electronic systems.

Gregory holds numerous patents and publications in technical and scientific magazines in Great Britain, Russia and the United States. Outside of work, Gregory's hobby is traveling, which is associated with his wife's business as a tour operator, and he publishes movies and pictures about his travels online.

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